FUN Exercise sheet 8. – date of submission 2018.04.26.

(89) Assume that T is an injective closed operator. Show that T^{-1} is also closed.

(90) (HW1) Let $DomT \subset X$ and $T : DomT \to Y$ be a closed linear operator, and let $C \subset X$ be a compact subset. Prove that $T(C)$ is closed in Y.

(91) Prove that in any normed space X and any vector $x \in X$ we have $\|x\| = \sup_{f \in X'} \frac{|f(x)|}{\|f\|}$.

(92) Prove that if $f(x) = f(y)$ for every $f \in X'$ then $x = y$.

(93) (HW2) Prove that if $T : DomT \to Y$ is a closed operator (where $DomT \subset X$) and $S \in B(X, Y)$ then $T + S$ is also a closed operator with $Dom(T + S) = DomT$.

(94) Let $x \in X$ and $g_x \in X''$ be defined by $g_x(f) = f(x)$. Prove that $\|g_x\| = \|x\|$.

(95) Let $T = T^* \in B(H)$ be a self-adjoint operator. Prove that if $\langle Tx, x\rangle = 0$ for all x, then $T = 0$ (even if H is a real Hilbert space).

(96) (HW3) Let H be a Hilbert space, and let $T : H \to H$ be a linear operator such that $DomT = H$, and $\langle Tx, y\rangle = \langle x, Ty\rangle$. Prove that T is automatically bounded (and thus self-adjoint).

(97) Let $T \in B(X, Y)$ and $x_n \xrightarrow{w} x \in X$. Prove that $Tx_n \xrightarrow{w} Tx$.

(98) Prove that $x_n \xrightarrow{w} x$ implies $\liminf \|x_n\| \geq \|x\|$.