FUN Exercise sheet 9. – date of submission 2018.05.08.

(99) (HW1) Let $X = (C[0,1], \|\cdot\|_\infty)$ and $v \in X$ be a fixed function. Let T be the multiplication operator by v, i.e. $Tx(t) = v(t)x(t)$. Find the spectrum of T.

(100) Let P be an orthogonal projection operator onto a closed subspace Y in a Hilbert space. Find the spectrum of P.

(101) (HW2) Let $T \in B(X,X)$. Prove that $\|R_\lambda(T)\| \to 0$ as $\lambda \to \infty$.

(102) Let $S, T \in B(X,X)$ and $\lambda \in \rho(S) \cap \rho(T)$. Prove that $R_\lambda(S) - R_\lambda(T) = R_\lambda(S)(T-S)R_\lambda(T)$.

(103) Prove that for any $T \in B(X,X)$ we have $r_\sigma(\alpha T) = |\alpha| r_\sigma(T)$, and $r_\sigma(T^n) = (r_\sigma(T))^n$.

(104) Show an example of a bounded linear operator $T : H \to H$ which is not self-adjoint, but has real spectrum.

(105) Let T be a multiplication operator on l^2 defined by $(x_1, x_2, \dots) \mapsto (\lambda_1 x_1, \lambda_2 x_2, \dots)$ for some bounded sequence (λ_n). What is the spectrum of T?

(106) Let f and g be two nonzero vectors in a Hilbert space, and let Q be the operator defined by $Q(x) = \langle x, f\rangle g$. Find the spectrum of Q.

(107) (HW3) Let U be a unitary operator in a Hilbert space. Show that if $\lambda \in \sigma(U)$ then $|\lambda| = 1$.

(108) Let T be unitary and self-adjoint. Prove that $\sigma(T) \subset \{-1, +1\}$.

(109) For $A \in B(X)$ the number $\lambda \in \mathbb{C}$ is an *approximate eigenvalue* of A if there is a sequence $\{v_n\}_{n\in\mathbb{N}} \subset X$ of unit vectors for which $\lim_{n\to\infty} \|Av_n - \lambda v_n\| = 0$. Prove that if λ is an approximate eigenvalue of A then $\lambda \in \sigma(A)$.