Calculus 1, 2018/19/I., topics for Part 1 of the Final exam

Part 1 of the Final exam will consist of:

• 12 definitions and/or theorems to state. Please find topics below. (36 points)

• 1 proof of a theorem that we have covered. Please find a list below. (20 points)

• 15 true/false questions about class material. These test your overall awareness of the class material. (30 points)

• 3 examples to give (e.g. give an example of a function or a sequence with certain prescribed properties). These also test your overall awareness of the class material. (14 points)

You are expected to be able to state the following definitions and theorems:

1. Definition of convergence of sequences (pg. 577).
2. Limit of the sum, difference, product and quotient of convergent sequences (pg. 579).
3. Sandwich theorem for sequences (pg. 580).
4. Definition of the supremum of an upper-bounded set of numbers (in class).
5. The nondecreasing sequence theorem (pg. 595).
6. Definition of convergence of series (pg. 585).
7. The nth term test for divergence of series (pg. 589).
8. Comparison test for nonnegative series (pg. 596).
9. Integral test for series (pg. 598).
10. Limit comparison test (pg. 600).
11. The ratio test (pg. 603).

12. The nth root test (pg. 606).

13. Leibniz-rule for alternating series (pg. 608).

14. Error estimation for alternating series (pg. 610).

15. Definition of absolute convergence (pg. 610).

16. The absolute convergence theorem (pg. 611).

17. Definition of power series (pg. 615).

18. Radius of convergence theorem – possible behaviour of power series (pg. 619).

19. Term-by-term differentiation of power series (pg. 620).

20. Multiplication of absolutely convergent series (pg. 621).

21. Definition of Taylor-polynomials and Taylor-series of a function (pg. 623, 624).

22. The Taylor-polynomial of order n has the same values of the first n derivatives as the function (pg. 625).

23. Taylor's theorem with remainder term (pg. 627).

24. Definition of limits of functions (in class with sequences, or pg. 115 with ε, δ).

25. The sandwich theorem for functions (pg. 89).

26. The sum, difference, product and quotient of limits (pg. 80, 95).

27. Definition of continuity of a function (pg. 103).

28. Algebraic properties of continuous functions (pg. 105).

29. The max-min theorem for continuous functions (pg. 108).

30. The intermediate value theorem for continuous functions (pg. 110).

31. Definition of differentiability of a functions (pg. 133).

32. If f is differentiable at x_0 then it is continuous at x_0 (pg. 137.)

33. The sum and difference rule for differentiation (pg. 144).

34. The product and quotient rule for differentiation (pg. 146, 147).

35. The chain rule (pg. 173).

36. Definition of linear approximation of a function (pg. 190).

37. Definition of local and global maxima and minima (pg. 226).

38. Rolle's theorem (pg. 228).

39. The mean value theorem (pg. 230).

40. Definition of monotonically increasing and decreasing functions (pg. 232).

41. The first derivative test for increasing and decreasing (pg. 233).

42. The definition of convex and concave functions (in class).

43. Second derivative test for convexity and concavity (pg. 237).

44. Definition of a point of inflection.

45. Antiderivatives can differ only by a constant (pg. 265).

46. The fundamental theorem of calculus (the Newton-Leibniz rule; pg. 309, 312).

47. Formulas for volume, surface and arc-length.

48. The integration-by-parts formula (pg. 529).

49. Integration by substitution formula (pg. 330).

50. Partial fraction decomposition of rational functions (pg. 551).

51. The substitution formulas for $z = \tan(\frac{x}{2})$.

You are expected to know the following proofs:

1. A monotonically increasing upper bounded sequence automatically converges.

2. The sandwich rule for the convergence of sequences.

3. The limit of the sum of convergent sequences is the sum of the limits.

4. The Leibniz-criterion for convergence of alternating series.

5. The nth term test for convergence of series.

6. The integral test for convergence of series.

7. The Taylor-polynomial of order n has the same values of the first n derivatives as the function.

8. A differentiable function is necessarily continuous.

9. The derivative of x^n is nx^{n-1}.

10. The derivative of $\sin x$ is $\cos x$.

11. Rolle's theorem.

12. The mean value theorem for the derivative of functions.

13. The first derivative test for increasing and decreasing (pg. 233).

14. Antiderivatives can differ only by a constant (pg. 265).

15. Functional inequalities: if $f(x_0) = g(x_0)$ and $f'(x) \leq g'(x)$ for all $x \geq x_0$ then $f(x) \leq g(x)$.

16. The Newton-Leibniz formula for continuous functions.