Calculus 2, 2018/19/II., topics for Part 1 of the Final exam

Part 1 of the Final exam will consist of:

• 12 definitions and/or theorems to state. Please find topics below. (36 points)

• 1 proof of a theorem that we have covered. Please find a list below. (19 points)

• 15 true/false questions about class material. These test your overall awareness of the class material. A correct answer is worth 3 points, an incorrect one -3 points, and a blank one 0 points. (45 points)

You are expected to be able to state the following definitions and theorems:

1. Norm of a vector, and the triangle inequality in $\mathbb{R}^p$ (pg. 2).

2. Scalar product and orthogonality of vectors in $\mathbb{R}^p$ (pg. 3).

3. The Cauchy-Schwartz inequality in $\mathbb{R}^p$ (pg. 2).

4. Convergence of a sequence of points in $\mathbb{R}^p$ (pg. 5).

5. Cauchy's criterion for convergence of sequences in $\mathbb{R}^p$ (pg. 6).

6. Bolzano-Weierstrass theorem in $\mathbb{R}^p$ (pg. 7).

7. Interior and boundary points of a set (pg. 8).

8. Definition of open sets, and their fundamental properties (pg. 11, Theorem 1.14).

9. Definition of closed sets and characterizing properties (pg. 12 Theorem 1.17).

10. Cantor's theorem (pg. 15, Theorem 1.25).

11. Definition of the limit of functions (pg. 21, Def. 1.33).

12. Transference principle (pg. 22, Theorem 1.38).

13. Definition of continuity and its characterization (pg. 24, Def. 1.42, Theorem 1.44).

14. Weierstrass min-max theorem (pg. 26, Theorem 1.51).

15. Definition of partial derivatives (pg. 31, Def. 1.56).

16. Definition of local maximum and minimum (pg. 32, Def. 1.59).

17. Local extrema and partial derivatives (pg. 32, Theorem 1.60).

18. Extrema of functions on a closed and bounded set (pg. 32, Theorem 1.61).

19. Differentiability (pg. 35, Def. 1.63).

20. Differentiability and partial derivatives (pg. 37, Theorem 1.67).

21. Continuous partial derivatives imply differentiability (pg. 39, Theorem 1.71).

22. Equation of a tangent plane (pg. 42, Def. 1.74).

23. Directional derivatives (pg. 43, Def. 1.76, Theorem 1.77).

24. Young's theorem (pg. 47, Theorem 1.82).

25. Taylor polynomial of order 2 (in class, or pg. 55, Def. 1.95, n=2)

26. Positive definite and negative definite quadratic forms (pg. 58, Def. 1.100).

27. Local extrema and the second derivative (pg. 58, Theorem 1.101).

28. Convex functions (pg. 60, Def. 1.104).

29. Convexity and the second derivative (pg. 61, Theorem 1.106).

30. The continuous image of a compact set is compact (pg. 69, Theorem 2.7).

31. Differentiability of an $f : \mathbb{R}^p \to \mathbb{R}^q$ function (pg. 70, Def. 2.11).

32. The derivative (Jacobian) matrix expressed with the partial derivatives (pg. 71, Theorem 2.13, Def 2.15).

33. The chain rule (pg. 74, Theorem 2.20).

34. Derivative of the inverse function (pg. 76. Theorem 2.26).

35. Banach fixed-point theorem (pg. 85, Theorem 2.36).

36. The implicit function theorem (pg. 89, Theorem 2.40).

37. Lagrange multiplier method (pg. 91, Theorem 2.44).

38. Jordan measurable sets (pg. 96, Def. 3.2).

39. Definition of null sets, i.e. sets of measure zero (pg. 99, Def. 3.8).

40. Characterization of measurability with the boundary of a set (pg. 99, Theorem 3.9).

41. The volume of a parallelepiped (pg. 113, Theorem 3.31).

42. Scaling of measure under a linear transformation (pg. 117, Theorem 3.35).

43. Calculating multivariate integrals by sections (pg. 138, Corollary 4.19).

44. Integration by substitution (pg. 140, Theorem 4.22).

45. Integration of a function along a curve (pg. 159, Theorem 5.8).

46. The Newton-Leibniz formula for line integrals in the presence of scalar potential (pg. 160, Theorem 5.11).

47. The existence of a scalar potential (pg. 173, Theorem 5.32).
48. The vectorial product of two vectors in $\mathbb{R}^3$ (pg. 184).
49. The surface area (pg. 185, Def. 5.41).
50. Integration along a surface (pg. 188, Def. 5.46).
51. The divergence theorem (pg. 192, Theorem 5.48, formula 5.34).
52. Stokes theorem (as in class).
53. Pointwise convergence of a function sequence (pg. 229, Def. 7.1).
54. Uniform convergence of a sequence of functions (pg. 230, Def. 7.5).
55. Cauchy criterion for uniform convergence (pg. 232, Theorem 7.9).
56. Uniform limit of continuous functions (pg. 233, Theorem 7.12).
57. Integration of uniform limit (pg. 235, Theorem 7.16).
58. Differentiation of uniform limit (pg. 236, Theorem 7.18).
59. Cauchy criterion for uniform convergence of function series (pg. 240, Theorem 7.25).
60. Weierstrass criterion for uniform convergence of function series (pg. 241, Theorem 7.27).
61. Fourier coefficients (pg. 271, Def 7.74).
62. The order of magnitude of Fourier coefficients of k-times continuously differentiable functions (pg. 276, Lemma 7.81)
63. Fourier series of twice continuously differentiable functions (pg. 276, Theorem 7.82).

You are expected to know the following proofs:

1. Bolzano-Weierstrass theorem (pg. 7).

2. Cantor's theorem (pg. 15).

3. Weierstrass min-max theorem (pg 26).

4. Banach fixed-point theorem (pg. 85).

5. The necessary condition for existence of a scalar potential (pg. 166, Theorem 5.17).

6. Cauchy criterion for uniform convergence. (pg 232).

7. Integration of uniform limit (pg. 235).

8. Weierstrass criterion for uniform convergence of function series (pg. 241).

9. The order of magnitude of Fourier coefficients of k-times continuously differentiable functions (pg. 276, Lemma 7.81)

10. Fourier series of twice continuously differentiable functions (pg. 276, Theorem 7.82).