Risk-Free Arbitrage Based on Public Information: An Example*

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Abstract

We document an arbitrage opportunity against a $200 million company that required only widely available public information and generated a guaranteed return of 25.6% in a few days. Less than $60,000 was invested into exploiting the opportunity. The likely reason is that while arbitrage was easy to check once recognized, it was difficult to spot.

1 Introduction

Arbitrage—the exploitation of mispricing for riskless or almost riskless profit—is one of the central concepts in finance. In a textbook example, identical securities trade at different prices in different markets, so an investor can sell the more expensive security and buy the cheaper one, making a profit now and being perfectly hedged looking forward. Because such an arbitrage strategy could generate arbitrarily large profits, in equilibrium the pricing inconsistency on which it is based cannot exist. While recent models of "limited arbitrage" have identified situations where mispricing can persist, even these models imply that completely riskless arbitrage that is based solely on public information would be exploited and hence *cannot* exist.[1]

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[1] For a summary of this research and a detailed discussion of arbitrage, see for instance Shleifer (2000). For examples of theories of limited arbitrage, see De Long, Shleifer, Summers, and Waldmann (1990) and Shleifer and Vishny (1997).

In this note we document an instance in which one firm, Szerencsejáték Rt. (SzjRt) of Hungary, created a striking arbitrage opportunity against itself, and survived it with flying colors. Our purpose is not to argue that arbitrage is a meaningless concept or that opportunities such as this are common. We merely provide a glaring example for a more simple and modest point: that just because a phenomenally good money-making opportunity exists, people may not recognize it. Since most economic and finance models are predicated on the assumption that people will spot any money-making opportunity, these models will mispredict behavior in this case and more generally fail to predict how fast arbitrage opportunities will be closed. Hence, finance models incorporating the cognitive limitations of individuals would seem useful.

SzjRt is Hungary's state-owned monopoly for lotteries and sports gambling, and with annual revenues topping $500 million it is one of the largest companies in the country. It cannot declare bankruptcy, and its promises to pay are backed by the government, which has never defaulted on an obligation. This means that the chance of SzjRt not honoring a bet is virtually nil. And in one week in 1998, a carefully constructed set of bets in one sports gambling game could generate a guaranteed return of over 25% in less than a week, fully at SzjRt's expense. The information necessary for carrying out the arbitrage was conspicuously posted in the 350 gambling parlors and countless post offices around the country, published in newspapers, and certainly read by the tens of thousands who played the game.

Figure 1 shows SzjRt's weekly profits in 1998 from the game in which arbitrage was possible. While Week 31 seems to be the obvious candidate for the arbitrage week, in that week the company merely got very-very unlucky. In fact, the arbitrage opportunity occurred in Week 26, in which SzjRt posted the year's *highest* profits from the game! Clearly, most people did not take advantage of this chance. As we argue below, while the strategy is relatively easy to understand ex post for anyone with a mathematical background, it was probably difficult to spot.[2] Indeed, while we have no individual-level data on the bets made, we are able to put an upper bound on the amount of money that could have been placed as part of an arbitrage strategy. We identify some wagers that

[2] Interested readers can—instead of reading our explanation—try to find the arbitrage opportunity themselves by opening and looking at (our translation of) what gamblers saw. We conjecture that even with the substantial benefits of a very high IQ and of the knowledge that there *is* an opportunity, readers will not find it immediately apparent. In fact, one author, although he was a regular player of the game at the time, failed to spot the opportunity.

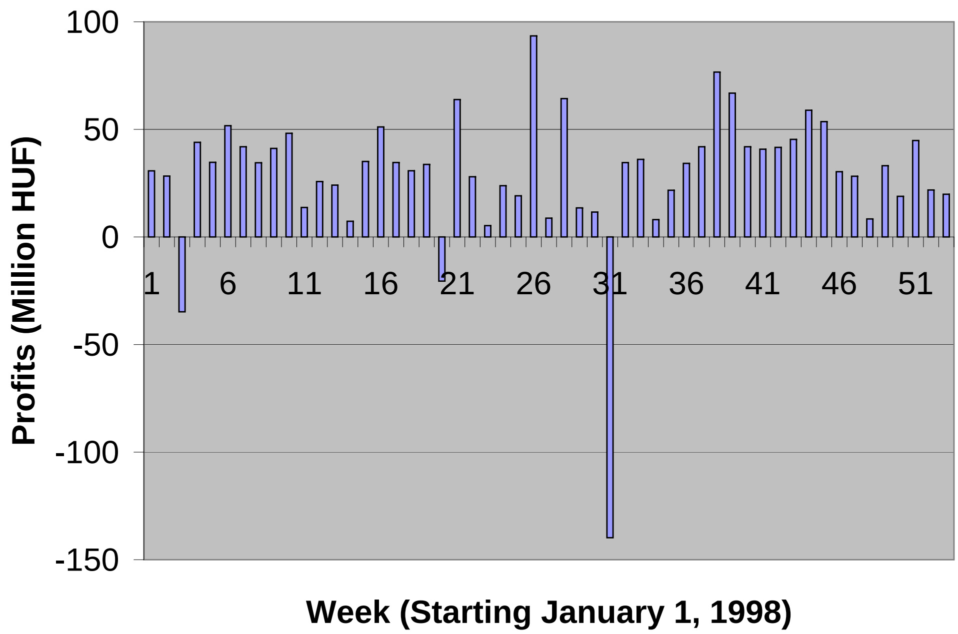

Figure 1: Szerencsejáték Rt's Weekly Profits From the Tippmix Game in 1998. (Note: The $-HUF exchange rate at the time was 210.)

had to be involved in arbitrage and look at summary statistics on these wagers. The upper bound is 12,511,000 HUF (about $59,500), a mere 5.46% of SzjRt's revenue and 11% of SzjRt's payout from the game that week.

2 Some Background

Szerencsejáték Rt. may be an unusual target for arbitrage, but if anything it is a safer bet for investors than most targets. The company is Hungary's state-owned monopoly provider of number-draw games, sports bets and prize draw tickets. With an annual revenue of just under 44 billion HUF (about $210 million) in 1998, it is one of the largest companies in the country. Its profit is absorbed into, and its potential losses are backed by, the state budget. Because SzjRt therefore has no option to go bankrupt, its promises to pay are more credible than those of most private companies, regarding which the question of arbitrage usually arises. In addition, while SzjRt reserves the right to cancel sports bets, the 1991 Act on the Organization of Gambling Operations allows it do so only when the actual sports event is canceled or there is a case of documentable corruption. Because the arbitrage opportunity is based on matches in soccer's biggest international event, the World Cup, there is essentially no chance that any of these would happen.

The relevant details of the game in question, Tippmix, were public information in any sense of the word commonly understood in economics and finance. The betting opportunities were published in SzjRt's sports-gambling newsletter *Sportfogdás* (circulation: 100,000), the sports daily *Nemzeti Sport* (which is read by most sports fans), on Teletext, at several hundreds of SzjRt outlets, at numerous commercial venues, and at almost all post offices in Hungary. Even considering how many people actually *read* the information—a measure that seems stricter than that typically applied for the publicness of information—details of Tippmix were widespread. Conservative estimates of SzjRt indicate that an average of at least 10,000 people played each week. Since revenue in our week of interest was way above the year's average (229,032,600 HUF versus 111,128,321 HUF), it is likely that the actual number of players was higher still. Furthermore, while Tippmix was targeted at a general audience, unpublished market studies of SzjRt suggest that it drew much interest from the wealthier, college-educated part of the population.

3 The Opportunity

In order not to overload the word "event," we abuse the word "match" instead, somewhat inappropriately calling any sports event a match, and reserving "event" for the statistical concept. Hence, for instance, a match being a tie is an event. Crucial to the arbitrage opportunity we analyze is that there may be multiple ways to subdivide outcomes of a match for wagering purposes. We call one such division a partition. For a Chicago-Dallas soccer match, for instance, Szerencsejáték Rt. might accept wagers on whether Chicago wins, Dallas wins, or the match is a tie, but also on whether Chicago wins by at least two goals, 1 goal, or does not win. Our interest will be in how to construct a *risk-free arbitrage* or simply *arbitrage*—a set of wagers that produces a positive return for any outcome of the matches in question. Relatedly, we define a *balanced betting strategy* as a

#	game	former	tie	latter
8.	Chicago-Dallas	1.50	3.00	4.00
9.	Ivanchuk-Kramnik	3.20	1.50	3.65

Table 1: Two Bettable Partitions from Week 27, 1998

betting strategy that yields 1 HUF in winnings in any contingency, and a *balanced arbitrage* is a balanced betting strategy that is also an arbitrage.

The arbitrage opportunity we analyze arose in the Tippmix game offered by SzjRt. Tippmix is a relatively complicated sports gambling game, as it gives individuals substantial flexibility to make "combined bets" putting multiple matches into a single wager. Hence, for example, in Week 27 in 1998 gamblers could combine bets on the outcomes of the Ivanchuk-Kramnik chess match in Germany and the Chicago-Dallas match above. Table 1 shows part of the two lines corresponding to the partitions for these matches in the list of 114 bettable partitions that week. Each event of each partition has an "odds" specifying the amount of money one can win when betting on that event alone. For example, the odds for Dallas winning the Chicago-Dallas game was 4, meaning that if an individual put 1HUF on this event and she got it right, she would have received 4HUF. If a person bets on multiple partitions at the same time, she wins money only if she guesses right on all partitions; but in that case, the multiplier for the combined event is the product of the odds of the individual events. Hence, for example, if someone put a "double bet" on Dallas and Ivanchuk winning and was right, she would have received $4 \times 3.2 = 12.8$ times her wager.

The following lemma will help us develop our argument:

Lemma 1. *An arbitrage involving a given set of partitions exists if and only if a balanced arbitrage involving the same set of partitions exists.*

Proof. The "if" part is trivial. To see the "only if" part, suppose there is an arbitrage opportunity. Take the contingency that yields the lowest winnings. Decrease wagers on other contingencies until all contingencies yield the same winnings. This clearly yields a balanced arbitrage. $\square$

SzjRt of course made sure that no single partition could be used for arbitrage. To see this, we

#	game	former	tie	latter
8.	Ivanchuk1-Kramnik1	3.20	1.50	3.65
9.	Ivanchuk2-Kramnik2	3.20	1.50	3.65

Table 2: Hypothetical Example with Logically Connected Bets

can apply Lemma 1 and calculate, for each partition, how much investment it takes to make sure one wins 1 HUF for any event in the partition. This is exactly the sum of the reciprocals of the events' multipliers; call it the sum-reciprocal. For both matches in Table 1, the sum-reciprocal is about 1.25. This means not only that there is no arbitrage opportunity for either partition, but also that SzjRt aims for a hefty profit from these partitions: about 20% of the wagers.

Consider now combined bets involving several partitions. It is crucial for our analysis to distinguish between logically "connected" and "independent" partitions.

Definition 1. *A set of partitions is logically independent if no combination of events in a subset of the partitions rules out an event in another partition.*

The inability to arbitrage single partitions carries over to wagers on multiple independent partitions:

Lemma 2. *An arbitrage that uses only logically independent partitions exists if and only if the product of the sum-reciprocals of the individual partitions is strictly less than 1. Hence, if such an arbitrage exists, an arbitrage involving a single partition also exists.*

Proof. Follows from the appendix. □

Intuitively, to generate a guaranteed 1 HUF when wagering on independent partitions, one has to put appropriate wagers on all combinations of elements in the partitions. The cost of doing so is exactly the product of the costs of balanced bets on the individual partitions. Hence, non-arbitrageable independent partitions cannot be combined into an arbitrage.

But in Week 26 in 1998, Szerencsejáték Rt. made a mistake: it allowed gamblers to bet on multiple highly *non*-independent partitions of the same match. To illustrate the implications in an extreme example, suppose that bettors could place wagers on two partitions both being determined by the outcome of the actual Ivanchuk-Kramnik match, as illustrated in the hypothetical Table 2. Then, to win 1HUF if Ivanchuk wins, it suffices to place a double wager of $1/(3.2 \times 3.2)$ on both Ivanchuk1 and Ivanchuk2 winning (recall that the odds from different partitions are multiplied). Extending this logic, to win 1HUF for any outcome of the match, it suffices to spend $1/(3.2 \times 3.2) + 1/(1.5 \times 1.5) + 1/(3.65 \times 3.65) \approx 0.62$. Hence, a bettor can make a riskless 61% return on her money! Intuitively, if partitions are drawn from different matches, to make sure one wins one must bet on all combinations of events. But if partitions are logically related because they derive from the same match, it may take much fewer wagers to cover all possible outcomes of the match.

The actual arbitrage opportunity was of course more complicated because the partitions were not identical, and because SzjRt had restrictions in place on the minimum number of partitions that had to be involved in the wagers. We can construct a balanced arbitrage strategy using the Argentina-Croatia game in Week 26 in the following way. Gamblers could bet on whether the game would be a win, tie, or loss for Argentina, as well as on what the score would be (0-0, 1-0, 0-1, etc., or "other"). Any exact score fully determines whether the game is a win, tie, or loss, so the two partitions are very closely linked logically. To construct an arbitrage, we combine each score with the appropriate summary outcome, and the "other" score with all three possibilities, covering all possible outcomes of the match. A difficulty is that for these highly arbitragable partitions gamblers had to place combined bets involving at least five partitions. But it just so happens that SzjRt allowed bets on another strongly related pair of partitions: whether Yugoslavia would win, tie, or lose its match with the United States, and whether it would win by at least 2 goals, 1 goal, or not win. As our fifth (or possibly sixth) partition, we use the number of goals scored by either Suker (a Croat) or Batistuta (an Argentine) or both, since if either Croatia or Argentina is scoreless, that automatically implies their player does not score. The details of this arbitrage strategy are in the appendix, where we show that it produced a risk-free return of 25.6% in just a few days.

The above was not the only balanced arbitrage opportunity in Week 26. SzjRt allowed bets both on whether Germany would win, tie or loose its match with Iran, and also on whether it would win by at least two goals, 1 goal or not win. As we show in the appendix, combining these two partitions with the aforementioned three or four related partitions on the Argentina-Croatia match creates a balanced arbitrage with a risk-free return of 20.2%. And combining the logically related partitions on all three matches yields a balanced arbitrage with a risk-free return of 22.3%. There were also infinitely many non-balanced arbitrage strategies that could be constructed from the games.[3]

4 Evidence on Responses

Even based on aggregate evidence, it is immediately apparent that exploitation of the arbitrage opportunity could not have been widespread. As shown in Figure 1, SzjRt realized a record profit from Tippmix during the week in question. While this record profit is partly due to record demand for Tippmix, it is also due to a 41% profit rate that was well above the year's average of 26%. In contrast, if everyone had followed the most profitable balanced arbitrage strategy, then SzjRt would have realized a loss of 25%.

Beyond these aggregate measures, our data allows us to give a rough upper bound on the amount of money that could have been invested into arbitrage strategies, even though we do not have data that would allow us to identify betting strategies at the individual level. For each bettable partition, we know the total amount of money that was spent in wagers involving that partition (including individual bets and combined bets of all sizes). Hence, we exploit that certain partitions had to be involved in any arbitrage strategy. As we have argued in Section 3, any arbitrage strategy must involve partitions that are logically connected. In Week 26 there were exactly three matches with logically connected partitions: the Yugoslavia-US, Argentina-Croatia, and Germany-Iran matches of the World Cup. To place wagers on any of these logically connected partitions bettors had to make at least five-fold combined bets. Based on simple but tedious calculations, we show in the appendix that any arbitrage had to involve the Argentina-Croatia match. And because combining

[3] Implementing one of these strategies involved modest but non-trivial transaction costs because it required filling out many tickets. For example, implementing the most profitable balanced arbitrage above would have required filling out 116 tickets, which we estimate would have taken one to two hours. This work was, however, largely a fixed cost: once the tickets were filled out, a bettor could increase the stakes of the bet in principle without limit.

this match with independent partitions (from which SzjRt took a 20% cut) would have eliminated the profitability of the wager, an arbitrage had to involve at least one of the other two matches.

Based on this very rough consideration, we get a very low upper bound for the amount of "smart money" in Tippmix. Consumers spent 5,235,000 HUF ($25,000) betting on whether Germany wins by 2 goals, 1 goal or does not win against Iran, and 7,276,000 HUF ($34,500) betting on whether Yugoslavia wins by 2 goals, 1 goal or does not win against the US. This means that no more than 12,511,000 HUF ($59,500), a mere 5.46% of the wagers in Tippmix, was spent on arbitrage strategies. Furthermore, our data suggests that much or most of even this money may not have been coming from arbitrageurs. In Week 26 the amount placed on "Belgium wins its soccer match by 2 goals/1 goal/does not win," a partition that is independent of all others, was 8,902,000 HUF ($42,500)—higher than the amount placed on either arbitrage partition of the same form. Similarly, in nearby weeks where there was no arbitrage opportunity, the amount of money placed on bets of the form "Team X wins its soccer match by 2 goals/1 goal/does not win" varied between 3% and 10% of revenue. A combined 5.46% on two such partitions is on the low side of this range of non-arbitrage demand.

5 Conclusion

Our note is only the case study of a single instance where market participants failed to exploit a very profitable arbitrage opportunity. In itself, it answers neither why most individuals failed to spot the opportunity, nor how common unexploited arbitrage opportunities are. To gain traction on these questions, a systematic analysis of a large number of arbitrage opportunities would be useful.

Appendix (Not Intended for Publication)

To prove the existence of a balanced arbitrage strategy we have to show that there exists a betting strategy which yields 1 HUF for sure and costs less than $1 HUF$. Before proving this claim, let us develop some notation and terminology. We denote by O_k^i the odds of the $i-th$ outcome of the $k-th$ partition; by C_k the cost of a balanced betting strategy on the $k-th$ partition P_k; by C^K the cost of a balanced strategy involving partitions $\{P_k\}_{k=1}^K$. Finally, we denote by $E^K(i,j,...,z)$ the event that corresponds to the intersection of the $i-th$ outcome of the first partition, the $j-th$ outcome of the second partition and so on up to the $z-th$ outcome of the $K-th$ partition. Given this notation the precise definition for two partition to be connected is that there exists i and j such that $E^2(i,j)=\emptyset$. Our next lemma shows how to calculate the cost of a balanced strategy for K independent partitions.

Lemma 3. *If all partitions $\{P_k\}_{k=1}^K$ are independent then the cost of a balanced strategy involving K different partitions is equal to the product of the cost of the balanced strategies on each partition alone.*

Proof. Since $C_1 = \sum_i 1/O_i^1$ it follows that $C^K = \sum_i^1 .... \sum_z^K 1/(O_i^1 * .....O_z^K) = \sum(1/O_i^1)...... \sum_z^K 1/(O_z^K) = \prod_i^k C_k$. □

This lemma implies that if partitions are independent and there is no arbitrage possibility on any single partition then there is no arbitrage possibility on multiple partitions and hence proves Lemma 2 in the text. As a corollary of this lemma it is also true that if we consider K partitions such that the first L is independent from the last $K-L$, then the cost of a balanced strategy is equal to the product of C^L the cost of an balanced strategy on the first L games C^L and the cost of a balanced strategy on the last $K-L$ partitions, C^{K-L}.

Corollary 1. *If we divide K partitions such that the first L is independent from the last $K-L$ then $C^K = C^L C^{K-L}$.*

If partitions are connected then the above lemma does not hold anymore since certain combination of outcomes are logically impossible and hence one does not need to bet money on such outcomes.

Corollary 2. $C^K = \prod_i^k C_k - C^D$ *where* $C^D = \{.\sum(1/(O_i^1 * ..... * O_z^K))$ *for all* $(i,j,...z)$ *such that* $E(i,j,...z) = \not\emptyset\}$

Having established these facts we can now turn to the demonstration of balanced arbitrage strategies.

Claim 1. *There are balanced arbitrage strategies.*

To prove this claim consider the following balanced arbitrage opportunity we mentioned in Section 3.

#	Partition	O_1	O_2	O_3
1	Argentina-Croatia	1.7	2.9	3.2
2	Suker will score: 0, 1, more goals	1.4	2.65	6.15
3	Batistuta will score: 0, 1, more goals	1.7	2.15	5
4	Yugoslavia-United States	1.15	4.2	7.3
5	Yugoslavia wins by: 2 goals, 1 goal, does not win	1.35	3.3	5
6.1	Argentina-Croatia: {0,0}	6.65		
6.2	Argentina-Croatia: {1,0}	4		
6.3	Argentina-Croatia: {0,1}	5.7		
6.4	Argentina-Croatia: {1,1}	4		
6.5	Argentina-Croatia: {2,0}	5		
6.6	Argentina-Croatia: {0,2}	10		
6.7	Argentina-Croatia: {2,1}	5.35		
6.8	Argentina-Croatia: {1,2}	10		
6.9	Argentina-Croatia: {2,2}	10		
6.10	Argentina-Croatia: {3,0}	11.45		
6.11	Argentina-Croatia: {0,3}	20		
6.12	Argentina-Croatia: {3,1}	8		
6.13	Argentina-Croatia: {1,3}	16		
6.14	Argentina-Croatia: ELSE	13.35		

Note that partitions 1 and 6 are connected since the outcome of partition 6 perfectly determines the outcome of partition 1 Furthermore, partitions $2-6$ and $3-6$ are also connected since if either

Argentina or Croatia scores no more than one goals then it already implies that Batistuta or Suker could not have scored more than 1 goal. In addition if the outcome of the Argentina-Croatia is $\{0, 0\}$ then we know that neither Batistuta nor Suker scored. Partitions 4 and 5 are also connected. If the United States does not loose against Yugoslavia then the outcome of partition 5 must be that Yugoslavia does not win. Hence combining these two games there are only four possible events and a balanced strategy costs $C^{Y,US} = 1/(1.15 * 1.35) + 1/(1.15 * 3.3) + 1/(4.2 * 5) + 1/(7.3 * 5) = 0.982\,64$.

To make calculations more transparent, we introduce the symbol S to stand for the cost of a balanced strategy based on partition 2 conditional on the event that Croatia scores no more than two goals. Since this implies that Suker scored less than 2 goals the cost is $S = 1/1.4 + 1/2.65 = 1.\,091\,6$. Similarly, we introduce the symbol B to stand for the cost of the balanced strategy based on partition 3, conditional on the fact that Argentina scores no more than two goals is $B = 1/1.7 + 1/2.15 = 1.\,053\,4$.

$C^*_{6.1} = 1/(6.65 * 2.9 * 1.4 * 1.7)$

$C^*_{6.2} = 1/(4 * 1.7 * 1.4)$

$C^*_{6.3} = 1/(5.7 * 3.2 * 1.7)$

$C^*_{6.4} = 1/(4 * 2.9) * B$

$C^*_{6.5} = 1/(5 * 1.7 * 1.4)$

$C^*_{6.6} = 1/(10 * 3.2 * 1.7)$

$C^*_{6.7} = 1/(5.35 * 1.7) * S$

$C^*_{6.8} = 1/(10 * 3.2) * B$

$C^*_{6.9} = 1/(10 * 2.9) * 1.25$

$C^*_{6.10} = 1/(11.45 * 1.7 * 1.4)$

$C^*_{6.11} = 1/(20 * 3.2 * 1.7)$

$C^*_{6.12} = 1/(8 * 1.7) * S$

$C^*_{6.13} = 1/(16 * 3.2) * B$

$C^*_{6.14} = 1/13.35 * 1.25^2$

The total cost for these four events is $C^{*}=\sum_{i=1}^{14}C_{6.i}^{*}=0.813\,31$. Since Szjrt required partition 6 be part of at least quintuple bets to show the existence of an arbitrage strategy we combine these partitions with partitions 4 and 5. By virtue of Lemma 3 the cost of an arbitrage strategy that bets on partitions $1-6$ and delivers 1 HUF for sure is given $C^{6}=C^{*}*C^{Y,US}=0.799\,19$.This means that there is a 25.1 percent risk-less return on this strategy. This proves our claim.

To show the other risk-free arbitrages strategies we mentioned in the text. consider the following two partitions:

#	Partition	O_1	O_2	O_3
7	Germany-Iran	1.1	4.7	8
8	Germany wins by: 2 goals, 1 goal, does not win	1.3	3.5	5.3

Since unless Iran looses Germany does not win the cost of a balanced strategy is $C^{Ger,Iran}=1/(1.1*1.3)+1/(1.1*3.5)+1/(4.7*5.3)+1/(8*5.3)=1.022\,8$ and a balanced strategy on partitions $1,2,3,6,7,8$ costs $C^{6'}=C^{*}*C^{Ger,Iran}=0.831\,85$. This strategy then yields a risk-free return of 20.2 percent. Furthermore, from Lemma 3 it follows that a balanced strategy on partitions $1-8$ costs $C^{8}=C^{*}*C^{Y,US}*C^{Ger,Iran}=0.817\,41$ and delivers a risk-free return of 22.3 percent.

Claim 2. *All arbitrage strategies have to involve partition* 6*. Any arbitrage strategy has to involve either partition* 5 *or partition* 8*.*

Note first that for all k $C_k>1.24$. It follows that there are no arbitrage strategies based on independent partitions. There were partitions other than those mentioned above that referred to the same match but just like partitions 1 and 3 none of them were connected.[4] This means that an arbitrage strategy must have contained at least two connected partitions out of partitions $1-8$.

Partitions $4-5$ are connected and a balanced strategy on a combined bet costs less than 1 HUF. These events, however, must have entered five-fold bets or more. Since $C^{Y,US}*1.24^{3}=1.873\,5$, combining these partitions with three independent ones precludes arbitrage. Even if we combine partitions $4,5$ and $7,8$ and a fifth independent one, however, the cost of a balanced strategy is

[4]To verify this claim please check the offer of bets for 26 in our supplmenetal material at www.econ.berkeley.edu/˜

at least $C^{Ger,Iran} * C^{Y,US} * 1.24 = 1.246\,2$. A similar argument shows that there is no risk-free arbitrage strategy that involves partitions $7-8$ but not 6.

To prove that any arbitrage strategy must have included either partition 5 or partition 8 consider the case of the cheapest risk-free betting strategy including partitions $1-3$ and 6. The cost of this strategy is $C^* = 0.813\,31$. Given the constraint that partition 6 could only be part of at least five-fold bets we need to add two more partitions to this strategy. Clearly adding two independent partitions would eliminate the positive risk-free return since $0.813\,31 * 1.24^2 = 1.250\,5$. The only option then is to add connected partitions, The only connected ones, however are partitions $4-5$ and $7-8$. This proves our claim.

References

De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann (1990): "Noise Trader Risk in Financial Markets," *Journal of Political Economy*, 98(4), 703–738.

Shleifer, A. (2000): *Clarendon Lectures: Inefficient Markets.* Oxford University Press.

Shleifer, A., and R. W. Vishny (1997): "The Limits of Arbitrage," *Journal of Finance*, 52(1), 35–55.