Programme on

"Mixing Flows and Averaging Methods"

April 4 – May 25, 2016

organized by

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Workshop 2

"Statistical properties of dynamical systems"

May 9 – 13, 2016

- **Monday, May 9, 2016**

  9:50 – 10:20 **Opening & Registration**

  10:20 – 10:50 *coffee / tea break*

  10:50 – 11:40 **Jean-Pierre Conze**

  11:50 – 12:40 **Jacopo de Simoi**

  12:40 – 14:40 *lunch break*

  14:40 – 15:30 **Marta Tyran-Kaminska**

  15:30 – 16:00 *break*

  16:00 – 16:50 **Sandro Vaienti**

- **Tuesday, May 10, 2016**

  9:30 – 10:20 **David Kelly**

  10:20 – 10:50 *coffee / tea break*

  10:50 – 11:40 **Yves Guivarc'h**

  11:50 – 12:40 **Alexey Korepanov**

  12:40 – 14:40 *lunch break*

  14:40 – 15:30 **Matthew Nicol**

  15:30 – 16:00 *break*

  16:00 – 16:50 **Gary Froyland**

- **Wednesday, May 11, 2016**

  9:30 – 10:20 **Imre Péter Tóth**

  10:20 – 10:50 *coffee / tea break*

  10:50 – 11:40 **Damien Thomine**

  11:50 – 12:40 **Poster session**

  Free afternoon

- **Thursday, May 12, 2016**

  9.30 – 10:20 **Dmitry Dolgopyat**

  10:20 – 10:50 *coffee / tea break*

  10:50 – 11:40 **Zemer Kosloff**

  11:50 – 12:40 **Françoise Pène**

  12:40 – 14:40 *lunch break*

  14:40 – 15:30 **Andrew Török**

  15:30 – 16:00 *break*

  16:00 – 16:50 **Tomas Persson**

- **Friday, May 13, 2016**

  9:30 – 10:20 **Leonid Bunimovich**

  10:20 – 10:50 *coffee / tea break*

  10:50 – 11:40 **Alexander Grigo**

  11:50 – 12:40 **Jon Aaronson**

All talks take place at the ESI, Boltzmann Lecture Hall!

Titles and abstracts

- **Jon Aaronson**: "Rational ergodicity of "discrepancy" skew products and the asymptotics of affine random walks."
  Abstract: It is not hard to show that if $(X(n) : n = 1, 2, ..)$ is given by a RAT (=random affine transformation) of form $X(n+1) = a(n+1)X(n) + b(n+1)$ where $\{(a(n), b(n)) : n = 1, 2, , , , \}$ are iidrvs taking values in $\{-1, +1\} \times R$ with $E(b) = 0$ and $E(b^2)$ finite, then $X(n)$ satisfies both central and local limit theorems. We'll establish a "weak, rough, local limit theoremfor certain non-stationary, multidimensional versions of this and use this to show rational ergodicity of "discrepancy" skew products $T : X = [0, 1) \times Z \to X$ by $T(x, z) = (x + A \ mod\, 1, z + D(x))$ where A is irrational and the "discrepancy function"$D = 1$ on $[0, 1/2)$ and $D = -1$ on $[1/2, 1)$.

  For each A there is a RAT sequence depending on the minus-signcontinued fraction expansion of A. For badly approximated A, this RAT sequence satisfies the "weak, rough, local limit theorem", whence bounded rational ergodicity of T_A.

  This is joint work with Michael Bromberg and Hitoshi Nakada. See arXiv:1603.07233

- **Leonid Bunimovich**: "Where and when orbits of the most chaotic systems prefer to go."
  Abstract: We consider the most chaotic deterministic and the most random stochastic systems .Then the curves of first hitting probabilities for elements of a Markov partition either are identical or intersect just ones. Among other things it allows to construct optimal towers.

- **Jean-Pierre Conze**: "Martingales and cumulants for algebraic $\mathbb{Z}^d$-actions on shift-invariant subgroups of $\mathbb{F}_p^{\mathbb{Z}^d}$"
  Abstract: Let (G, μ) be a compact abelian group with its Haar measure μ, $(T^\ell)_{\ell \in \mathbb{Z}}$ a $\mathbb{Z}^d$-action on G by automorphisms or endomorphisms, f a regular function on G.

  If (Z_k) is a r.w. on $\mathbb{Z}^d$, a question is that of a quenched CLT for the sums $\sum_{k=0}^{n-1} T^{Z_k(\omega)} f$. We consider also the ergodic sums $\sum_{\ell \in K_n} T^\ell f$, for (K_n) an increasing family of rectangles.

  We will focus on a family of non connected groups G, namely the shift-invariant subgroups of $\mathbb{F}_p^{\mathbb{Z}^d}$. A result of C. Cuny, J. Dedecker and D. Volný, in the spirit of M. Gordin's martingale reduction, can be used for the ergodic sums. The case of random walks (with a moment of order 2) is based on estimates related to polynomial relations.

  This gives a class of examples for the CLT for $\mathbb{Z}^d$-actions which are not mixing of all orders, including F. Ledrappier's example (1978).

- **Dmitry Dolgopyat**: "Local Limit Theorem for Markov chains."
  Abstract: We prove a local limit theorem for inhomogenuous Markov chains with finite state space under conditions similar to the conditions of Dobrushin's Central Limit Theorem. This is a joint work with Omri Sarig.

- **Gary Froyland**: "Hölder continuity of Oseledets splittings for semi-invertible cocycles"
  Abstract: For Hölder continuous cocycles over an invertible, Lipschitz base, we establish the Hölder continuity of Oseledets subspaces on compact sets of arbitrarily large measure. This extends a result of Araújo, Bufetov, and Filip by considering possibly semi-invertible cocycles, which in addition may take values in the space of compact operators on a Hilbert space.

- **Alexander Grigo**: "Applications of billiard-like systems"
  Abstract: In this talk we will consider billiard-like models that arise in certain models of gas-like interacting particle systems. The aim of the talk will be to present how one can implement perturbation theory, particularly averaging theory, to compute statistical properties. In particular, we will be interested in studying transport properties.

- **Yves Guivarc'h**: "On Fréchet's law and some extreme value properties for multivariate affine random walk."

Abstract: We consider a general multivariate affine stochastic recursion and we assume a natural geometric condition which implies the existence of an unbounded stationary solution. We show that the corresponding stationary process satisfies extreme value properties of classical type,with a non trivial extremal index.The proofs are based on a spectral gap property for the affine random walk.

- **David Kelly**: "Fast-slow systems with chaotic noise."
  Abstract: We will discuss the rough path approach to deterministic homogenization. The talk will start with an introduction of the problem of deterministic homogenization and the rough path methods employed. We will also focus on details that were not covered in the mini-course from the previous week, namely how the methods can be extended to discrete time systems and non-product systems. This is based on joint work with Ian Melbourne.

- **Alexey Korepanov**:"Homogenization for families of skew products"
  Abstract: We consider families of fast-slow skew product maps of the form

  $$x_{n+1} = x_n + \varepsilon^2 a_\varepsilon(x_n, y_n) + \varepsilon b_\varepsilon(x_n) v_\varepsilon(y_n), \quad y_{n+1} = T_\varepsilon y_n,$$

  where T_ε is a family of nonuniformly expanding maps, v_ε is of mean zero and the slow variables x_n lie in $\mathbb{R}^d$. Under an exactness assumption on b_ε (automatically satisfied in the cases $d = 1$ and $b_\varepsilon \equiv I_d$), we prove convergence of the slow variables to a limiting stochastic differential equation (SDE) as $\varepsilon \to 0$.

  Our results include cases where the family of fast dynamical systems T_ε consists of intermittent maps, unimodal maps (along the Collet-Eckmann parameters) and Viana maps.

  Similar results are obtained also for continuous time systems

  $$\dot{x} = \varepsilon^2 a_\varepsilon(x, y, \varepsilon) + \varepsilon b_\varepsilon(x) v_\varepsilon(y), \quad \dot{y} = g_\varepsilon(y).$$

  Here, as in classical Wong-Zakai approximation, the limiting SDE is of Stratonovich type $dX = \bar{a}(X)\,dt + b_0(X) \circ dW$ where $\bar{a}$ is the average of a_0 and W is a d-dimensional Brownian motion. This is a joint work with Z. Kosloff and I. Melbourne.

- **Zemer Kosloff**: "Intersection local times for random walks and some questions in ergodic theory"
  Abstract: I will talk on a joint work with George Deligiannidis where we use limit theorems on the intersection local times in order to derive two properties, namely Folner property for the range of recurrent 2d (or 1d Cauchy) random walksänd älmostëquidistribution of its local times. The latter two are applied to the study of growth of information of random walks in random sceneries.

- **Matthew Nicol**: "A dichotomy for the distribution of returns times and extremes for uniformly hyperbolic systems."
  (joint with M. Carvalho, A. C. M. Freitas. J. M . Freitas and M. Holland)
  Abstract: We consider return times to small balls centered at points p in the phase space of a hyperbolic toral automorphism $T : \mathbb{T}^2 \to \mathbb{T}^2$. We show that return times are Poisson to small balls centered at non-periodic points and compound Poisson for small balls centered at periodic points and give the distributional limit in terms of the period of p and the metric. A closely related result is that if F is a regular function of a Euclidean-type distance to p that is strictly maximized at p and p is non-periodic then the corresponding time series $\{F \circ T^i.\}$ exhibits extreme value statistics corresponding to an iid sequence of random variables with the same distribution function as F and with extremal index one. If however F is maximized at a periodic orbit q then the corresponding time-series exhibits extreme value statistics corresponding to an iid sequence of random variables with the same distribution function as F but with extremal index not equal to one. We give a formula for the extremal index in this case.

- **Françoise Pène**: "Quantitative recurrence for slowly mixing hyperbolic systems."
  Abstract: We are interested in the counting process of visits to a small around the origin, and more precisely in its behaviour as the radius of the ball goes to 0. We prove the convergence in distribution of this process (under a suitable time normalization) to the standard Poisson process. This is a joint work with Benoît Saussol.

- **Tomas Persson**: “Shrinking targets.”
  Abstract: There has recently been some interest in various shrinking target problems in dynamical systems. One considers a dynamical system T and investigate when the orbit of a point x enters a shrinking neighbourhood of a point y infinitely many times. For instance, one can choose a random point x and try to determine the size of the set of points y for which this happens when the neighbourhoods around y shrink at a fixed speed. I will talk about this and in particular about a recent work with Michał Rams, based on another work with Henry Reeve.

- **Jacopo de Simoi**: “Birkhoff conjecture and spectral rigidity of planar convex domains.”
  Abstract: Dynamical billiards constitute a very natural class of Hamiltonian systems: in 1927 George Birkhoff conjectured that, among all billiards inside smooth planar convex domains, only billiards in ellipses are integrable. In this talk we will prove a version of this conjecture for convex domains that are sufficiently close to an ellipse of small eccentricity. We will also describe some remarkable relation with inverse spectral theory and spectral rigidity of planar convex domains. Our techniques can in fact be fruitfully adapted to prove spectral rigidity among generic (finitely) smooth axially symmetric domains which are sufficiently close to a circle. This gives a partial answer to a question by P. Sarnak.

- **Damien Thomine**:“Potential kernel, hitting probabilities and limit distributions.”
  Abstract: Abelian extensions of hyperbolic dynamical systems are a class of measure-preserving transformations which are useful to understand diffusion phenomena. The simplest examples are random walks on $\mathbb{Z}$ or $\mathbb{Z}^2$; this class also includes interesting examples with a more dynamical flavour, such as Lorentz' gases, which are $\mathbb{Z}$ or $\mathbb{Z}^2$-periodic convex billiard.
  The goal of this work is to transpose in the context of $\mathbb{Z}^d$ extensions a result which is known for random walks, namely, the link between the potential kernel of the random walk (which is an elementary solution of the Poisson's equation associated with the transition kernel of the random walk) and the probability that an excursion hits a given point. We shall also show how to interpret this link using the limit distribution of a well-chosen process.
  Joint work with Françoise Pène (Université de Bretagne Occidentale).

- **Imre Péter Tóth**: “Equidistribution for standard pairs in planar dispersing billiard flows.”
  Abstract: Correlation decay for planar dispersing billiard flows has been shown to be at least stretched exponential by Chernov in 2007, and indeed exponential by Baladi, Demers and Liverani in 2015. These results concern observables with some Hölder regularity, and can be translated into equidistribution statements for absolutely continuous initial measures with correspondingly regular densities. However, for applications in averaging problems, it is often convenient to have equidistribution theorems for initial measures that are much less regular: in particular, one that is concentrated on a single unstable curve. Such measures are called standard pairs. In this talk I present an approximation technique with which we obtain strong equidistribution theorems for standard pairs, using the above results about correlation decay.

  This is joint work with Péter Bálint, Péter Nándori and Domokos Szász.

- **Andrew Török**: “Almost sure convergence of maxima for chaotic dynamical systems.”
  (joint work with M. P. Holland and M. Nicol)
  Abstract: Suppose $(f, \mathcal{X}, \nu)$ is a probability measure preserving dynamical system and $\phi : \mathcal{X} \to \mathbb{R}$ is an observable with some degree of regularity. We investigate the maximum process $M_n := \max\{X_1, \ldots, X_n\}$, where $X_i = \phi \circ f^i$ is a time series of observations on the system. When $M_n \to \infty$ almost surely, we establish results on the almost sure growth rate, namely the existence (or otherwise) of a sequence $u_n \to \infty$ such that $M_n/u_n \to 1$ almost surely. The observables we consider will be functions of the distance to a distinguished point $\tilde{x} \in \mathcal{X}$. Our results are based on the interplay between shrinking target problem estimates at $\tilde{x}$ and the form of the observable (in particular polynomial or logarithmic) near $\tilde{x}$. We determine where such an almost sure limit exists and give examples where it does not. Our results apply to a wide class of non-uniformly hyperbolic dynamical systems, under mild assumptions on the rate of mixing, and on regularity of the invariant measure.

- **Marta Tyran-Kaminska**:“Convergence to bivariate Levy processes and limit theorems in infinite ergodic theory.”
  Abstract: Generalized central limit theorem are known to hold for dynamical systems with an infinite ergodic measure which induce Gibbs-Markov maps with regularly varying tails, see for example the work by Damien Thomine (2014). We show how one can deduce such distributional limit theorems from functional limit theorems for bivariate observables of the induced dynamical systems.
- **Sandro Vaienti**:“On a few statistical properties of intermittent sequential systems”