DVR 0065528

Programme on
"Mixing Flows and Averaging Methods"
April 4 – May 25, 2016

organized by

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Mini-Courses 2

"Statistical properties, averaging and homogenization"

May 2 – 6, 2016

- **Monday, May 2, 2016**

  09:00 – 09:30 **Opening & Registration**

  09:30 – 11:00 **Viviane Baladi & Mark Demers**
  Exponential mixing for two-dimensional Sinai billiard flows, Lecture 1

  11:00 – 11:30 *coffee / tea break*

  11:30 – 13:00 **Sébastien Gouëzel**
  Quantitative estimates for non–uniformly hyperbolic dynamical systems, Lecture 1

  13:00 – 15:00 *lunch break*

  15:00 – 16:30 **Viviane Baladi & Mark Demers**
  Exponential mixing for two-dimensional Sinai billiard flows, Lecture 2

- **Tuesday, May 3, 2016**

  09:30 – 11:00 **Jacopo de Simoi**
  Limit theorems and statistical properties of some fast-slow systems, Lecture 1

  11:00 – 11:30 *coffee / tea break*

  11:30 – 13:00 **Sébastien Gouëzel**
  Quantitative estimates for non–uniformly hyperbolic dynamical systems, Lecture 2

  13:00 – 15:00 *lunch break*

  15:00 – 16:30 **Viviane Baladi & Mark Demers**
  Exponential mixing for two-dimensional Sinai billiard flows, Lecture 3

- **Wednesday, May 4, 2016**

  09:30 – 11:00 **Jacopo de Simoi**
  Limit theorems and statistical properties of some fast-slow systems, Lecture 2

  11:00 – 11:30 *coffee / tea break*

  11:30 – 13:00 **David Kelly**
  Rough paths and fast-slow systems, Lecture 1

  13:00 – 15:00 *lunch break*

  15:00 – 16:30 **Viviane Baladi & Mark Demers**
  Exponential mixing for two-dimensional Sinai billiard flows, Lecture 4

- **Thursday, May 5, 2016**

  Ascension day, public holiday in Austria, no talks

- **Friday, May 6, 2016**

  09:30 – 11:00 **Jacopo de Simoi**
  Limit theorems and statistical properties of some fast-slow systems, Lecture 3

  11:00 – 11:30 *coffee / tea break*

  11:30 – 13:00 **Sébastien Gouëzel**
  Quantitative estimates for non–uniformly hyperbolic dynamical systems, Lecture 3

  13:00 – 15:00 *lunch break*

  15:00 – 16:00 **Mikko Stenlund**
  Quasistatic dynamical systems

  16:00 – 16:15 *break*

  16:15 – 17:45 **David Kelly**
  Rough paths and fast-slow systems, Lecture 2

All talks take place at the ESI, Boltzmann Lecture Hall!

- **Viviane Baladi & Mark Demers:** Exponential mixing for two-dimensional Sinai billiard flows

  Abstract: Sinai billiards are natural chaotic dynamical systems which are notoriously difficult to study because of their singularities (arising from "grazing orbits"). Their first ergodic properties were obtained by Sinai over forty years ago. In 1998, L.-S. Young obtained exponential mixing for the (discrete-time) Sinai billiard map. In 2007, Chernov and Melbourne proved independently that the Sinai billiard flow mixes faster than any polynomial. The minicourse will consist in a presentation of an outline of our recent proof (with C. Liverani) that two-dimensional finite-horizon Sinai billiard flow mix in fact exponentially fast. The proof is based on a study of the spectrum (i.e., the Ruelle resonances) of the billiard transfer operator on a suitable space of anisotropic distributions.
  In the first two lectures, V. Baladi will present the result, give the functional analytic background (defining the anisotropic norm), prove the Lasota-Yorke estimates allowing to control the essential spectral gap and state the Dolgopyat bound, thus providing all the ingredients which, combined, give the spectral gap result. In the remaining two lectures, M. Demers will prove the Dolgopyat cancellation bound, after constructing and giving precise Lipschitz estimates on the "approximate unstable foliations" which are one of the key new tools in this work.

- **Sébastien Gouëzel:** Quantitative estimates for non–uniformly hyperbolic dynamical systems

  Abstract: TBA

- **David Kelly:** Rough paths and fast-slow systems

  Abstract: Fast-slow systems often exhibit effective low dimensional behavior in the limit of large time-scale separation. The approximation of slow variables in a coupled fast-slow system is known as averaging or homogenization (depending on the context). Averaging/Homogenization typically arises due to rapid oscillations in the fast variables only making statistical contributions to the slow variables over larger time scales.
  When the fast variables are stochastic and exhibit strong mixing properties, the derivation of homogenized equations has been well studied since the 60s. When the fast variables are deterministically chaotic, the mixing properties are typically much weaker and the homogenization is far less straight forward.
  The purpose of this mini-course is to introduce rough path theory and show how it can be used in the study of homogenization for deterministic systems. The key insight is that rough path theory allows for the construction of continuous maps from 'noise' appearing in the equation to solution of slow variables. In particular one can lift homogenization results for the noise alone to results for the solution, via the continuous map.
  The outline of the course will be roughly as follows. We will discuss stochastic integrals, and how they arise from limits of random and deterministic sums. We will introduce the rough path approach to stochastic integration and stochastic differential equations. Finally we will show how this theory can be applied to the homogenization of fast-slow systems, both stochastic and deterministic.

- **Jacopo de Simoi:** Limit theorems and statistical properties of some fast-slow systems

  Abstract: In this minicourse we study a large class of fast-slow systems, which can be regarded as a deterministic realization of Wentzell-Friedlin systems. We will show how Large Deviation Principle and Local Central Limit theorem with error bounds allow to prove (virtually) optimal bounds for the rate for the decay of correlations. The proofs of the limit theorems requires a blend of spectral methods and standard pairs techniques, which I plan to describe in some detail. Having established the limit theorems, existence of physical measures and the sharp bounds on exponential decay of correlation follow from a more flexible version of the coupling argument (originally introduced by Dolgopyat and Chernov) which will be illustrated in depth.

- **Mikko Stenlund:** Quasistatic dynamical systems

  Abstract: We discuss an abstract framework for modeling quasistatic non-equilibrium processes arising in physics. After that we present some limit results for a concrete model.