Test 1

Exercise 1 (7p)**.** *Solve the following ODE:*

$$x(y' - 2x^3 \ln x) = 4y.$$

Exercise 2 (7p)**.** *Solve the following ODE:*

$$y' + 6y^2 = \frac{5}{x^2}.$$

Exercise 3 (7p)**.** *Solve the following IVP:*

$$\begin{aligned} 2x + 2yy' &= 0 \\ y(-5) &= 12. \end{aligned}$$

Exercise 4 (7p)**.** *Solve the following ODE:*

$$x^2 y \, \mathrm{d}x + x^3(y^3 + \ln x) \, \mathrm{d}y = 0.$$

Exercise 5 (7p)**.** *Solve the following IVP:*

$$\left(\frac{y}{x^2y^2 + 1} + 2x \, \mathrm{d}x\right) + \left(\frac{x}{x^2y^2 + 1} + 1\right) \mathrm{d}y = 0, \quad y(0) = 2.$$

Exercise 6 (5p+5p+5p)**.** *Find the approximation of the solution to the IVP*

$$\begin{aligned} y' &= 2y \\ y(0) &= 2. \end{aligned}$$

at $x = 2$ by the explicit Euler's method with constant step size $h = \frac{1}{4}$. Find the exact solution using the method of undetermined coefficients or the method of Taylor series. Finally find the solution using successive approximations.

Multiplier formula:

$$\frac{\varphi'(z)}{\varphi(z)} = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M\frac{\partial z}{\partial x} - N\frac{\partial z}{\partial x}}.$$