

CORRELATION DECAY IN BILLIARDS

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Summary: Motion of a point particle in a planar periodic array of circular scatterers is considered. Motion outside the scatterers is uniform. The scatterers are either rigid (“hard” billiards), or described by an axis-symmetric potential (“soft”). An utmost important feature of this system is the *non-continuity* of the collision-to-collision dynamics, that is, the presence of *singularities*.

Hyperbolicity and ergodicity of these systems have long been investigated (see eg. [5],[4]). Recently, the problem of exponential decay correlations (EDC) was also solved for the 2D hard case ([6],[3]). Our aim was to extend this to high dim. and soft systems using the method in [3].

For high dim. systems, 4 out of five conditions required by the theorem of [3] were checked. On the other hand, the 5-th condition was found to be greatly problematic due to the complicated structure of singularities (see [1]). For 2D soft systems we managed to apply the method in [3] and obtain EDC for a reasonably defined subclass of the systems discussed in [4].

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