Probability 1
CEU Budapest, fall semester 2016
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Homework sheet 4 – due on 07.11.2013 – and exercises for practice

4.1 **(homework)** *Poisson approximation of the binomial distribution.* Fix $0 < \lambda \in \mathbb{R}$. Show that if X_n has binomial distribution with parameters (n, p) such that $np \to \lambda$ as $n \to \infty$, then X_n converges to $Poi(\lambda)$ weakly.

4.2 **(homework)** Let X be uniformly distributed on $[-1; 1]$, and set $Y_n = nX$.

a.) Calculate the characteristic function ψ_n of Y_n.

b.) Calculate the pointwise limit $\lim_{n\to\infty} \psi_n(t)$, if it exists.

c.) Does (the distribution of) Y_n have a weak limit?

d.) How come?

4.3 Durrett [1], Exercise 3.3.1

4.4 Durrett [1], Exercise 3.3.3

4.5 Durrett [1], Exercise 3.3.9

4.6 **(homework)** Durrett [1], Exercise 3.3.10. Show also that independence is needed.

4.7 Durrett [1], Exercise 3.3.11

4.8 **(homework)** Durrett [1], Exercise 3.3.12

4.9 Durrett [1], Exercise 3.3.13

4.10 **(homework)** Let $X_1, X_2, \dots$ be i.i.d. random variables with density (w.r.t. Lebesgue measure) $f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$. (So they have the Cauchy distribution.) Find the weak limit (as $n \to \infty$) of the average

$$\frac{X_1 + \dots + X_n}{n}.$$

Warning: this is not hard, but also not as trivial as it may seem. Hint: a possible solution is using characteristic functions. Calculating the characteristic function of the Cauchy distribution is a little tricky, but you can look it up.

4.11 Durrett [1], Exercise 3.3.20

4.12 Durrett [1], Exercise 3.4.4

4.13 Durrett [1], Exercise 3.4.5

4.14 Durrett [1], Exercise 3.6.1

4.15 Durrett [1], Exercise 3.6.2

References

[1] Durrett, R. *Probability: Theory and Examples.* Cambridge University Press (2010)