

Probability 1
CEU Budapest, fall semester 2017

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Working time: 120 minutes $\approx \infty$

Every question is worth 10 points. Maximum total score: 30.

1. Today, Alice rolls a fair die, and she will be sad if the result is not 6. Tomorrow she tries at most twice, and she will only be sad if neither are 6. Every day she tries: on day n she rolls the die until she gets a 6, but at most n times – and she will be sad if she doesn't manage to roll a 6.

What is the probability that she will be sad on infinitely many days?

2. Bob takes a long walk, making n steps. At each step, independently of the others, he falls with probability $\frac{3}{n}$. Let X_n be the number of falls. Find the weak limit of X_n as $n \rightarrow \infty$.
3. Is there a sequence of random variables X_n such that
 - a.) $X_n \Rightarrow 0$, $0 \leq X_n \leq 1$, but $\mathbb{E}X_n \rightarrow 1$?
 - b.) $X_n \rightarrow \infty$ almost surely, but $\mathbb{E}X_n \rightarrow 0$?

If not, why not? If yes, give an example!

4. We toss a fair coin infinitely many times. For $n = 1, 2, 3, \dots$ let $X_n = 1$ if the n th and the $n + 1$ st tosses are both heads, and 0 if not. Let $S_n = X_1 + \dots + X_n$. Show that $\sqrt{\frac{S_n}{n}} \Rightarrow \frac{1}{2}$.