

**Probability 1**  
**CEU Budapest, fall semester 2013**  
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**Homework sheet 5 – due on 28.10.2013 – and exercises for practice**

- 5.1 (**homework**) Let  $X_1, X_2, \dots$  be i.i.d. random variables with density (w.r.t. Lebesgue measure)  $f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$ . (So they have the Cauchy distribution.) Find the weak limit (as  $n \rightarrow \infty$ ) of the average

$$\frac{X_1 + \dots + X_n}{n}.$$

*Warning: this is not hard, but also not as trivial as it may seem. Hint: a possible solution is using characteristic functions. Calculating the characteristic function of the Cauchy distribution is a little tricky, but you can look it up.*

- 5.2 Durrett [1], Exercise 3.3.20
- 5.3 (**homework**) Durrett [1], Exercise 3.4.4
- 5.4 Durrett [1], Exercise 3.4.5
- 5.5 (**homework**) Durrett [1], Exercise 3.6.1
- 5.6 Durrett [1], Exercise 3.6.2

## References

- [1] Durrett, R. *Probability: Theory and Examples*. Cambridge University Press (2010)