

## **M4. PROBABILITY 1**

**Course Coordinator:** Imre Péter Tóth

**No. of Credits:** 3, and no. of ECTS credits: 6

**Time Period of the course:** Fall Semester

**Prerequisites:** basic probability

**Course Level:** introductory MS

### **Brief introduction to the course:**

The course introduces the fundamental tools in probability theory.

### **The goals of the course:**

The main goal of the course is to learn fundamental notions like Laws of Large Numbers, martingales, and Large Deviation Theorems.

### **The learning outcomes of the course:**

By the end of the course, students are enabled experts on the topic of the course. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

### **More detailed display of contents:**

Week 1-2: Review of basic notions of probability theory. Measure-theoretic language. Some famous problems and paradoxes.

Week 3: Different types of convergence for random variables. Borel-Cantelli lemmas.

Week 4: Laws of Large Numbers. The method of characteristic functions in proving weak convergence: the Central Limit Theorem.

Week 5-6: Conditional expectation with respect to a sub-sigma-algebra. Martingales. Some martingale convergence and optional stopping theorems.

Week 7: Applications of martingales: Galton-Watson branching processes. Asymptotic results. Birth and death process.

Week 8-9: Probabilistic methods in combinatorics. Second moment method, Lovász Local Lemma.

Week 10: Some large deviation theorems, Azuma's inequality.

Week 11-12: Random walks on the integers. Construction and basic properties of Brownian motion.

**References:**

1. R. Durrett: Probability. Theory and Examples. 4th edition, Cambridge University Press, 2010.
2. N. Alon, J. H. Spencer: The Probabilistic Method. 3<sup>rd</sup> edition, Wiley, 2008.