**Probability 1**
**CEU Budapest, fall semester 2014**
Imre Péter Tóth
**questions of the oral final exam**

On the exam each student will get two of the questions below, randomly but not independently. Beside, they will get a calculation exercise similar to those in the exercise sheets. Of course, during the conversation, other questions related to the course may come up.

1. Probability space, probability, random variable, distribution of a random variable. Construction of random variables.
2. Expectation of a random variable, expectation of a probability distribution. Properties of the expectation.
3. Pairwise independence, mutual independence. Independence and expectation.
4. Borel-Cantelli lemmas.
5. Strong law of large numbers for i.i.d. random variables with a finite 4th moment.
6. Weak convergence: alternative definitions and their equivalence. Relation to strong convergence and convergence in probability.
7. Weak law of large numbers: $L^2$ case and general case.
8. Characteristic function of a random variable. Characteristic function of a probability distribution. Definitions and basic properties.
9. Characteristic functions: relation to the expectation and higher moments.
10. Characteristic functions and weak convergence.
11. Central limit theorem (for i.i.d. random variables).
12. Conditional expectation with respect to a $\sigma$-algebra. Definition, existence and uniqueness.
13. Jensen's inequality for conditional expectations.
14. Martingales: definition, basic properties, examples. Discrete stochastic integral.
15. The upcrossing inequality and the martingale convergence theorem.
16. Branching processes and the martingale convergence theorem.
17. Doob's optional stopping theorem. Application to the ABRACADABRA problem.
18. Wiener process (Brownian motion): motivation, definition, basic properties.
19. Paul Lévi's construction of the Wiener process.