Probability 1
CEU Budapest, fall semester 2014
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Homework sheet 5 – due on 13.11.2014 – and exercises for practice

5.1 Let X be uniformly distributed on $[-1;1]$, and set $Y_n = nX$.

a.) Calculate the characteristic function ψ_n of Y_n.

b.) Calculate the pointwise limit $\lim_{n\to\infty} \psi_n(t)$, if it exists.

c.) Does (the distribution of) Y_n have a weak limit?

d.) How come?

5.2 Show that if $X_n \Rightarrow X$ and $f : \mathbb{R} \to \mathbb{R}$ is continuous, then $f(X_n) \Rightarrow f(X)$.

5.3 **(homework)** Durrett [1], Exercise 3.3.1. *(Hint: $Re\phi = \frac{\phi+\bar{\phi}}{2}$; $|\phi|^2 = \phi\bar{\phi}$.)*

5.4 Durrett [1], Exercise 3.3.3

5.5 Durrett [1], Exercise 3.3.9

5.6 Durrett [1], Exercise 3.3.10. Show also that independence is needed.

5.7 **(homework)** Durrett [1], Exercise 3.3.11

5.8 Durrett [1], Exercise 3.3.12

5.9 **(homework)** Durrett [1], Exercise 3.3.13

5.10 Let $X_1, X_2, \dots$ be i.i.d. random variables with density (w.r.t. Lebesgue measure) $f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$. (So they have the Cauchy distribution.) Find the weak limit (as $n \to \infty$) of the average

$$\frac{X_1 + \cdots + X_n}{n}.$$

Warning: this is not hard, but also not as trivial as it may seem. Hint: a possible solution is using characteristic functions. Calculating the characteristic function of the Cauchy distribution is a little tricky, but you can look it up.

5.11 Durrett [1], Exercise 3.3.20

5.12 Durrett [1], Exercise 3.4.4

5.13 **(homework)** Durrett [1], Exercise 3.4.5 *(Hint: Use exercise 5.2 and Durrett [1], Exercise 3.2.14).*

5.14 Durrett [1], Exercise 3.6.1

5.15 Durrett [1], Exercise 3.6.2

References

[1] Durrett, R. *Probability: Theory and Examples, 4th edition.* Cambridge University Press (2010)