Probability 1
CEU Budapest, fall semester 2014
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Homework sheet 2 – due on 09.10.2014 – and exercises for practice

2.1 Let χ be the counting measure on $\mathbb{N}$. Calculate $\int_{\mathbb{N}} f \, d\chi$ if $f : \mathbb{N} \to \mathbb{R}$ is given by

a.) $f(k) := \frac{1}{2^k}$

b.) $f(k) := \frac{1}{k}$

c.) $f(k) := \frac{(-1)^k}{k}$

2.2 **(homework)** Let χ be the counting measure on $\mathbb{N}$ and let the measure μ be absolutely continuous with respect to χ, with density $f(k) := q^k p$, where $p \in (0, 1)$ and $q = 1 - p$. Define $X : \mathbb{N} \to \mathbb{R}$ as $X(k) := k$.

a.) Calculate $\int_{\mathbb{N}} X \, d\mu$.

b.) Calculate $\int_{\mathbb{N}} X^2 \, d\mu$.

2.3 Let μ be a measure on $\mathbb{R}$ which has density $f(x) := x^2$ with respect to Lebesgue measure. Let ν be a measure on $\mathbb{R}$ which has density $g(x) := \sqrt{x}$ with respect to μ. Calculate $\nu([1, 3])$.

2.4 **(homework)** Let the random variable X have density

$$f(x) = \begin{cases} 2e^{-2x} \text{ if } x > 0 \\ 0 \text{ if not} \end{cases} ,$$

with respect to Lebesgue measure on $\mathbb{R}$.

a.) Show that this f is indeed the density (w.r.t. Lebesgue) of a probability distribution.

b.) Let $Y := X^2$. Show that Y is also absolutely continuous w.r.t. Lebesgue measure and find its density.

2.5 *Usefulness of the linearity of the expectation.* A building has 10 floors, not including the ground floor. On the ground floor, 10 people get into the elevator, and every one of them chooses a destination at random, uniformly out of the 10 floors, independently of the others. Let X denote the number of floors *on which the elevator stops* – i.e. the number of floors that were chosen by at least one person. Calculate the expectation and the variance of X. *(hint: First notice that the distribution of X is hard to calculate. Find a way to calculate the expectation and the variance without that.)*