

Probability 1
CEU Budapest, fall semester 2013
Imre Péter Tóth

Homework sheet 4 – due on 14.10.2013 – and exercises for practice

4.1 (**homework**) For real numbers a_1, a_2, a_3, \dots define the infinite product $\prod_{k=1}^{\infty} a_k$ as

$$\prod_{k=1}^{\infty} a_k := \lim_{n \rightarrow \infty} \prod_{k=1}^n a_k,$$

whenever this limit exists.

Let p_1, p_2, p_3, \dots satisfy $0 \leq p_k < 1$ for all k . Show that $\prod_{k=1}^{\infty} (1 - p_k) > 0$ if and only if $\sum_{k=1}^{\infty} p_k < \infty$.

(Hint: estimate the logarithm of $(1 - p)$ with p .)

4.2 Durrett [1], Exercise 3.3.1

4.3 Durrett [1], Exercise 3.3.3

4.4 Durrett [1], Exercise 3.3.9

4.5 (**homework**) Durrett [1], Exercise 3.3.10. Show also that independence is needed.

4.6 Durrett [1], Exercise 3.3.11

4.7 (**homework**) Durrett [1], Exercise 3.3.12

4.8 Durrett [1], Exercise 3.3.13

References

[1] Durrett, R. *Probability: Theory and Examples*. Cambridge University Press (2010)