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$$K \Psi_n(t) =$$

$$\sqrt{2} \int_0^t s \sin\left(\frac{\pi(2n-1)}{2} s\right) ds +$$

$$\sqrt{2} \int_t^1 t \sin\left(\frac{\pi(2n-1)}{2} s\right) ds$$

$$\int_0^t s \sin\left(\frac{\pi(2n-1)}{2} s\right) ds =$$

integrate by parts

$$- \frac{2}{\pi(2n-1)} \left[s \cdot \cos\left(\frac{\pi(2n-1)}{2} s\right) \right]_0^t +$$

$$\frac{2}{\pi(2n-1)} \int_0^t \cos\left(\frac{\pi(2n-1)}{2} s\right) ds =$$

$$- \frac{2}{\pi(2n-1)} t \cdot \cos\left(\frac{\pi(2n-1)}{2} t\right) + \frac{4}{\pi^2(2n-1)^2} \sin\left(\frac{\pi(2n-1)}{2} t\right)$$

$$\int_t^1 \sin\left(\frac{\pi(2n-1)}{2} s\right) ds = \frac{2}{\pi(2n-1)} \cdot \cos\left(\frac{\pi(2n-1)}{2} t\right)$$

putting these together we get: $K \Psi_n = \lambda_n \Psi_n \quad \square$