

Tools of modern probability, fall 2017

- 1) o and O -notation, asymptotic equivalence
- 2) Gaussian integrals
- 3) Polar coordinates in higher dimensions, surface of hyperspheres
- 4) Almost Gaussian integrals, Laplace's method
- 5) Euler gamma function, Stirling's approximation
- 6) Application: de Moivre-Laplace central limit theorem (CLT)
- 7) Measure space, probability space. Push-forward of measures. Distribution of random variables
- 8) Integral, expectation. Integration by substitution. Expectation of random variables. Densities of measures. Sums of series and Riemannian integrals as special cases of the (Lebesgue) integral.
- 9) Characteristic functions of random variables, characteristic functions of probability distributions. Characteristic functions and sums of independent random variables.
- 10) Exchanging the integral and the limit: monotone convergence theorem, dominated convergence theorem, Fatou lemma.
- 11) Application: differentiability of the characteristic function.
- 12) Product space, product measure. Exchanging integrals: Fubini's theorem
- 13) Hilbert spaces – Riesz representation theorem
- 14) Absolute continuity. Radon-Nikodym theorem.
- 15) Composition of measures and kernels. Decomposition of measures, conditional measure, factor measure.
- 16) Conditional expectation of random variables. Existence, uniqueness.
- 17) Jensen's inequality for conditional expectations.
- 18) Existence of weak limits -- vague convergence, Cantor's diagonal argument. Tightness and weak convergence.