The substitution $\frac{y^2}{2} = a(x-m)^2$ suggested in the exercise means $x = m \pm \frac{1}{\sqrt{2a}} y$, so we can (and should) choose a sign: let's choose $\oplus$, so
$x := m + \frac{1}{\sqrt{2a}} y$. (Choosing $\ominus$ would also do.)
This is now a bijection $\mathbb{R} \leftrightarrow \mathbb{R}$, $dx = \frac{1}{\sqrt{2a}} dy$,

So $$\int_{-\infty}^{\infty} e^{-a(x-m)^2} dx = \int_{-\infty}^{\infty} e^{-\frac{y^2}{2}} \frac{1}{\sqrt{2a}} dy \overset{\text{known}}{=\!=} \sqrt{2\pi} \frac{1}{\sqrt{2a}} = \sqrt{\frac{\pi}{a}}$$ □