Let c_n denote the $(n-1)$-dimensional surface volume of the <u>unit</u> sphere in $\mathbb{R}^n$.

We calculate the distribution function using spherical symmetry: $f(\underline{v}) = \frac{1}{\sqrt{2\pi}^n} e^{-\frac{|v|^2}{2}}$, so

$$F_E(z) = \mathbb{P}(E \le z) = \mathbb{P}\left(\tfrac{m}{2} v^2 \le z\right) = \mathbb{P}\left(|v| \le \sqrt{\tfrac{2z}{m}}\right) =$$

let $z > 0$

$$= \int_{B_{\sqrt{\frac{2z}{m}}}(0)} f(v_1, \dots, v_n)\, dv_1 \dots dv_n \xrightarrow{\text{spherical}}_{\text{symmetry}} \int_0^{\sqrt{\frac{2z}{m}}} c_n r^{n-1} \frac{1}{\sqrt{2\pi}^n} e^{-\frac{r^2}{2}}\, dr$$

$B_{\sqrt{\frac{2z}{m}}}(0) \leftarrow$ ball of radius $\sqrt{\frac{2z}{m}}$ around 0

$$\xrightarrow[x = \frac{r^2}{2};\ r = \sqrt{2x}]{\text{substitute}} \int_0^{z/m} c_n \sqrt{2x}^{\,n-1} \frac{1}{\sqrt{2\pi}^n} e^{-x} \frac{1}{\sqrt{2x}}\, dx = \frac{c_n}{2\pi^{n/2}} \int_0^{z/m} x^{\frac{n}{2}-1} e^{-x}\, dx$$

$dr = \frac{1}{\sqrt{2x}}\, dx$

normalizing constant, clearly $= \dfrac{1}{\int_0^\infty x^{\frac{n}{2}-1} e^{-x}\, dx} = \dfrac{1}{\Gamma\left(\frac{n}{2}\right)}$

So the density is

$$f_E(z) = F_E'(z) = \frac{1}{\Gamma\left(\frac{n}{2}\right)} \left(\frac{z}{m}\right)^{\frac{n}{2}-1} e^{-\frac{x}{m}},$$

So ~~E has~~ E has $\boxed{\text{Gamma distribution}}$

with shape parameter $\boxed{s = \frac{n}{2}}$

and rate parameter $\boxed{\lambda = \frac{1}{m}}$

$$\boxed{f_E(z) = \begin{cases} \frac{1}{\Gamma(s)} \lambda^{s-1} z^{s-1} e^{-\lambda z}, & \text{if } z > 0 \\ 0, & \text{if not} \end{cases}}$$