

I. $f_n(x) := n \mathbb{1}_{(0, \frac{1}{n})}(x)$, so $f_n(x) \rightarrow 0 \quad \forall x$ $f(x) := 0$
but $\int_X f_n d\mu = n \cdot \frac{1}{n} = 1 \not\rightarrow 0$.

II. The same f_n works.

III. No such example exists: if $f_n(x) \rightarrow 0$ for $\forall x \in \{1, 2, \dots, N\}$,
then clearly $\int_X f_n d\mu = \sum_{i=1}^N |f_n(i)| \rightarrow 0$ as well.

IV. $f_n(x) := \frac{1}{n}$ for $\forall x \in \{1, 2, \dots\}$. Then $f_n(x) \xrightarrow{n \rightarrow \infty} 0 =: f(x)$
 $\forall x$,

but $\int_X f_n d\mu = \sum_{i=1}^{\infty} \frac{1}{n} = \infty \quad \forall n$.