I. $f_n(x) := \sqrt{n}\,\mathbb{1}_{(0,\frac{1}{n})}$, $f \equiv 0$, so $f_n \xrightarrow{L^1} f = 0$

because $\int_X |f_n|\,d\mu = \sqrt{n}\cdot\frac{1}{n} = \frac{1}{\sqrt{n}} \to 0$,

but $\int_X |f_n|^2 d\mu = n\cdot\frac{1}{n} = 1 \not\to 0$, so $f_n \overset{L^2}{\not\to} 0$.

II. The same f_n works.

III. No such example exists: ~~if $f_n(i) \to$~~

if $\sum_{i=1}^{N} |f_n(i) - f(i)| \xrightarrow{n\to\infty} 0$, then $|f_n(i) - f(i)| \xrightarrow{n\to\infty} 0$ for $\forall i$,

and then $\sum_{i=1}^{N} |f_n(i) - f(i)|^2 \xrightarrow[n\to\infty]{\boxed{N<\infty}!!} 0$ as well.

IV. Again, no example: if $\sum_{i=1}^{\infty} |f_n(i) - f(i)| \to 0$, then it is < 1 for n big enough $\Rightarrow \forall i\ |f_n(i) - f(i)| < 1 \Rightarrow |f_n(i) - f(i)|^2 < |f_n(i) - f(i)|$

$\Rightarrow \sum_{i=1}^{\infty} |f_n(i) - f(i)|^2 \le \sum_{i=1}^{\infty} |f_n(i) - f(i)| \to 0$ $\square$