We already knew from d.) that $\varphi_\mu'(t) = \int_{\mathbb{R}} ixe^{itx}\,d\mu(x)$,

So let's check that this is continuous.

Let $t_n \to t \in \mathbb{R}$, then let $f_n, f: \mathbb{R} \to \mathbb{C}$ be

$$f_n(x) = ixe^{it_nx} \xrightarrow{n\to\infty} ixe^{itx} =: f(x),$$

so $\varphi_\mu'(t_n) = \int_{\mathbb{R}} f_n\,d\mu$ and $\varphi_\mu'(t) = \int_{\mathbb{R}} f\,d\mu$.

Now the <u>dominated convergence thm</u> ensures that

$\int_{\mathbb{R}} f_n\,d\mu \longrightarrow \int_{\mathbb{R}} f\,d\mu$, because $|f_n(x)| = |x|$

so $g(x) := |x|$ is a dominating fn for $\forall n$,

and it is integrable: $\int_{\mathbb{R}} g\,d\mu = \int_{\mathbb{R}} |x|\,d\mu < \infty$

by assumption: $\mathbb{E}\nu \in \mathbb{R}$ means exactly

that $\int_{\mathbb{R}} x_+\,d\mu < \infty$ and $\int_{\mathbb{R}} x_-\,d\mu < \infty$. $\square$