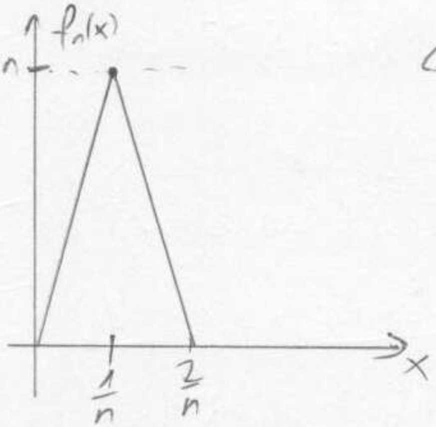


Clearly $f_n(x) \to 0$ for $\forall x \in \mathbb{R}$

$$\int_0^1 f_n(x)\,dx = \text{Area}(\Delta) = 1 \text{ for all } n \geq 2$$

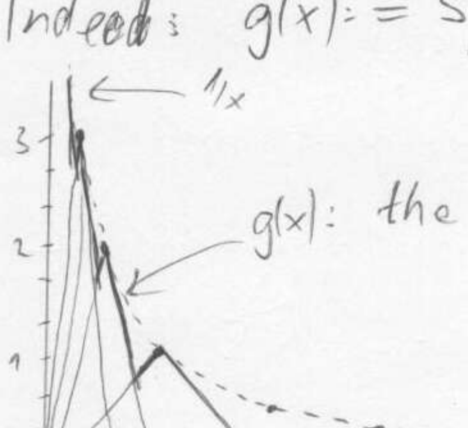
$$\Rightarrow \lim_{n\to\infty} \int_0^1 f_n(x)\,dx = 1 \neq 0 = \int_0^1 \lim_{n\to\infty} f_n(x)\,dx.$$

$\boxed{\text{Monotone convergence:}}$ the convergence is <u>not</u> monotoneous, so the thm says nothing.

[Alternatively: since $\int \lim \neq \lim \int$, the theorem says that the convergence <u>can not</u> be monotoneous]

$\boxed{\text{Dominated convergence:}}$ $\int \lim \neq \lim \int$, so there <u>can not</u> be an integrable dominating function.

Indeed: $g(x) := \sup_n f_n(x)$ is <u>not</u> integrable:



$g(x)$: the smallest possible candidate $\geq \frac{1}{2x}$

so $\int_0^\infty g(x)\,dx = \infty$ ☹

$\boxed{\text{Fatou lemma}}$ holds, since $f_n \geq 0$, and indeed

$\lim \int \geq \int \lim$ ✓✓