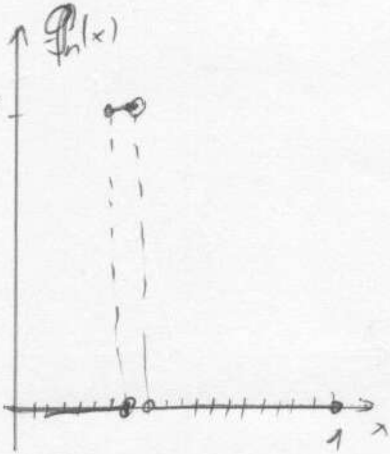

For every k, as l goes from 0 to 2^{k-1}

| so n goes from 2^k to $2^{k-1}-1$,

$$g_n(x) = \begin{cases} 1 & \text{for exactly one } l \\ 0 & \text{for all the others,} \end{cases}$$

So $\forall x \in [0,1]$ $g_n(x) = \begin{cases} 1 & \text{for infinitely many } n\text{-s} \\ 0 & \end{cases}$

$\Rightarrow \lim\limits_{n\to\infty} g_n(x)$ $\boxed{\text{does not exist}}$

$\Rightarrow$ Dominated convergence thm says <u>nothing</u>

Monotone convergence thm says <u>nothing</u>

The Fatou lemma <u>holds</u>, since $g_n \geq 0$

And indeed, $\liminf\limits_{n\to\infty} g_n(x) = 0$, and

$$\liminf_{n\to\infty} \int_0^1 g_n(x)\,dx \overset{n=2^k+1}{=\!=} \liminf \frac{1}{2^k} = 0$$

so $\liminf \int \geq \int \liminf$ ✓