

Midterm I.

Mathematics A3 in English for Civil Engineering students

Imre Péter Tóth, October 12, 2011

1. (6 points) Find the general solution of the differential equation

$$x^2 y' = 1 + y^2.$$

2. (6 points) Solve the Cauchy problem

$$xy' + y = \ln x \quad , \quad y(1) = 0.$$

3. (6 points) Find the equilibrium solutions of the following autonomous first-order differential equation, and characterize their stability.

$$y' = y^2(y - 3).$$

4. (6 points) Find the general solution of the differential equation

$$y'' + y = -2 \sin x.$$

5. (6 points) Find the general solution of the differential equation

$$y'' = -2(y')^2 \tan y.$$

Solutions

1. Separable. $y = \tan(C - \frac{1}{x})$
2. First order linear (inhomogeneous, with variable coefficients). $y = \ln x - 1 + \frac{1}{x}$. (The general solution of the diff.eq. is $y = \ln x - 1 + \frac{C}{x}$, and using the initial condition gives $C = 1$.)
3. There are two equilibrium solutions, 0 and 3. Both are unstable. (Actually 0 is semi-stable.)
4. Second order linear (inhomogeneous, with constant coefficients and special right hand side). The solution is $y = A \cos x + B \sin x + x \cos x$. (The solution of the homogeneous part is $Y_{\text{hom,gen}} = A \cos x + B \sin x$, and the good test function for the particular solution is $y_p = Cx \cos x + Dx \sin x$, because of resonance.)
5. Special second order equation, incomplete, with x missing. The solution is $y = \arctan(Cx + D)$.