

Math A3 English, practice class 1, autumn 2010

1.1 Give the general and a particular solution of the equation

$$y' = \frac{y}{x}.$$

1.2 Solve the separable differential equation

$$y' = e^{x+y}.$$

1.3 During radioactive decay, the amount $y(t)$ of the radioactive material decreases, with the rate of decrease being proportional to the amount of material still left. Find the coefficient of this proportionality, if the half-life of the material is 100 (seconds) – that is, the initial amount of the material is reduced to half during this time. What is the formula for the $y(t)$ function, if the initial quantity of the material was 1 (microgram)?

1.4 Find the solution of the Cauchy problem

$$y' = \frac{e^x}{y+1} \quad ; \quad y(0) = 1.$$

1.5 a.) A solid body moving with *low* speed, left alone in some medium (e.g. air, water) is slowing down with an acceleration proportional to the speed. How does the speed behave as a function of time? How about the position as a function of time? Does the body ever stop? Does it get infinitely far away?

b.) What happens, if the body, moving vertically, is also subject to gravity? Can we still answer these questions? How are the results modified?

1.6 a.) A solid body moving with *high* speed, left alone in some medium (e.g. air, water) is slowing down with an acceleration proportional to the *square of the speed*. How does the speed behave as a function of time? How about the position as a function of time? Does the body ever stop? Does it get infinitely far away?

b.) What happens, if the body, moving vertically, is also subject to gravity? Can we still answer these questions? How are the results modified?

Remark: The precise meaning of ‘low’ and ‘high’ speed in these models is a delicate issue in Physics.

1.7 Find the general and a particular solution of the separable equation

$$y' = \frac{1 + 2e^y}{e^y x \ln(x)}.$$

1.8 Solve the scary looking equation

$$(e^{-2y} - e^{-y})y' = \frac{e^{x-y} + e^{-x-y}}{e^y + 1}.$$

1.9 Solve the following homogeneous (in its variable) initial value problem (Cauchy problem):

$$x + y - xy' = 0 \quad ; \quad y(1) = 1$$

1.10 Find all the solutions of the equation

$$xe^{y/x} + y = xy'.$$

1.11 For which functions $y(x)$ does the homogeneous (in its variable) differential equation

$$xy' = y(\ln(y) - \ln(x))$$

hold?