Math A3 English, practice class 3, autumn 2010

3.1 Solve the following initial value problems, and also find the limit $\lim_{t\to\infty} y(t) =?$

a.) $y'' + 5y' + 6y = 0, \quad y(0) = 2,\ y'(0) = 3$

b.) $y'' + y' - 2y = 0, \quad y(0) = 1,\ y'(0) = 1$

c.) $y'' + 4y' + 3y = 0, \quad y(0) = 2,\ y'(0) = -1$

d.) $y'' + 8y' - 9y = 0, \quad y(1) = 1,\ y'(1) = 0$

3.2 Consider the following initial value problem:

$$y'' + 5y' + 6y = 0, \quad y(0) = 2,\ y'(0) = \beta,$$

where $\beta > 0$.

a.) Solve the initial value problem.

b.) As a function of β, find the coordinates (t_m, y_m) of the maximum point of the solution.

c.) Give the smallest possible value of β, for which $y_m \geq 4$.

d.) Describe the behaviour of t_m and y_m as $\beta \to \infty$.

3.3 Using the Euler formula, give the value of the following powers (with complex exponents) in the algebraic form $a + bi$ $(a, b \in \mathbb{R})$.

a.) e^{-3+6i}

b.) e^{1+2i}

c.) $e^{i\pi}$

d.) 2^{1-i}

3.4 Solve the following intial value problems:

a.) $16y'' - 8y' + 145y = 0, \quad y(0) = -2,\ y'(0) = 1$

b.) $y'' + 4y = 0, \quad y(0) = 1,\ y'(0) = 1$

c.) $y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0,\ y'(\pi/2) = 2$

d.) $y'' + 2y' + 2y = 0, \quad y(\pi/4) = 2,\ y'(\pi/4) = -2$

3.5 Solve the following intial value problems:

a.) $y'' - y' + 0.25y = 0, \quad y(0) = 2,\ y'(0) = 1/3$

b.) $9y'' - 12y' + 4y = 0, \quad y(0) = 2,\ y'(0) = -1$

c.) $9y'' + 6y' + y = 0, \quad y(0) = -1,\ y'(0) = 2$

d.) $y'' + 4y' + 4y = 0, \quad y(-1) = 2,\ y'(-1) = 1$

3.6 Consider the following initial value problem:

$$4y'' + 12y' + 9y = 0, \quad y(0) = 1,\ y'(0) = -4.$$

a.) Solve the initial value problem and sketch the solution on the interval $0 \leq t \leq 5$.

b.) Find the zeroes of the solution.

c.) Find the coordinates (t_0, y_0) of the minimum point of the solution.

d.) Change the second initial value to $y'(0) = \beta$, and give the solution as a function of β. Now find the critical value of β that separates those solutions, which are always positive, from those that become negative.