Math A3 English, practice class 4, autumn 2010

4.1 Find the general solution of the following differential equations with the test function method.

a.) $y'' - 3y' - 4y = 3e^{2t}$

b.) $y'' - 3y' - 4y = 2\sin t$

c.) $y'' - 3y' - 4y = -8t\cos(2t)$

d.) $y'' - 3y' - 4y = 3e^{2t} + 2\sin t - 8t\cos(2t)$

e.) $y'' - 3y' - 4y = 2e^{-t}$

4.2 Find the general solution of the following differential equations.

a.) $y'' - 2y' - 3y = 3e^{2t}$

b.) $y'' + 2y' + 5y = 3\sin(2t)$

c.) $y'' - 2y' - 3y = -te^{-t}$

d.) $y'' + 2y' + y = 2e^{-t}$

4.3 Solve the intial value problem:

$$y'' + 4y' = t^2 + 3t \quad , \quad y(0) = 0\,,\; y'(0) = 2.$$

4.4 Find the general solution of the following differential equation:

$$y'' + 4y = 3\csc t \qquad (\csc t = 1/\sin t).$$

4.5 Solve the following differential equations with the method of variation of the parameter (variation of the constant), and also with the test function method:

a.) $y'' - 5y' + 6y = 2e^{t}$

b.) $4y'' - 4y' + y = 16e^{t/2}$.

4.6 Find the general solution of the differential equation

$$y'' + y = \tan t, \quad 0 < t < \frac{\pi}{2}.$$

4.7 Find the general solution of the differential equation

$$4y'' + y = 2\sec(t/2), \quad -\pi < t < \pi \quad (\sec t = 1/\cos t).$$

4.8 Consider the following differential equation:

$$t^2y'' - t(t+2)y' + (t+2)y = 2t^3, \quad t > 0.$$

Check that tha functions

$$Y_1 = t, \quad Y_2 = te^t$$

form a fundamental solution of the correspoding homogeneous equation $t^2y'' - t(t+2)y' + (t+2)y = 0$. After this, find the general solution of the original inhomogeneous equation.