Math A3 English, practice class 5, autumn 2010

5.1 Solve the following ititial value problem:

$$y'' = \frac{1}{\sqrt{1-x^2}} \quad , \quad y(0) = 3\,,\, y'(0) = 1$$

5.2 If an elastic beam is subjet to bending moment which is proportional to the function $f(x)$ (along the cross section of the beam with the abscissa x), then the shape of the beam's median line can be calculated from the differential equation

$$\frac{y''}{(1+(y')^2)^{3/2}} = f(x).$$

Give the shape of the beam if the bending moment is given by

$$f(x) = 1 - x$$

and the initial conditions are

$$y(0) = y'(0) = 0.$$

5.3 a.) Solve the differential equation of free oscillation without external force and friction

$$my'' + ky = 0$$

as a special second order equation where y' does not appear.

b.) Solve the same equation as a linear second order equation with constant coefficients, using the characteristic polynomial method we learned two weeks ago.

c.) Show that the two results are the same.

5.4 Find the general solution of the following differential equations. It is enough to give the soultion in an implicite form.

a.) $(y')^2 + 2yy'' = 0$

b.) $y'' = \frac{1}{4\sqrt{y}}$

c.) $yy'' + (y')^2 = 1$

5.5 Solve the following second order differential equation:

$$xy'' - y' = x^3.$$

5.6 Solve the following differential equations:

a.) $2x\cos y + [2y\cos y - (x^2+y^2)\sin y]y' = 0$

b.) $x\,\mathrm{d}\,y + y\mathrm{d}\,x = 0$

c.) $\frac{x}{x^2+y^2}y' = \frac{y}{x^2+y^2}$

d.) $2x(\sin y + 1) + x^2\cos y \cdot y' = 0$