Problem Sheet - First practical course

1. A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy? Assume that the sample space S is given by $S = [(b, b), (b, g), (g, b), (g, g)]$, and all outcomes are equally likely. ((b, g) means for instance that the older child is a boy and the younger child a girl.)

2. Suppose we toss two fair dice. Let E_1 denote the event that the sum of the dice is six and F denote the event that the first die equals four. Show that E_1 and F are not independent. Let E_2 be the event that the sum of the dice equals seven. Is E_2 independent of F?

3. Assume that an urn contains 5 red balls, 3 white balls and 7 green balls. Draw 3 balls from the urn without replacing the already drawn balls to the urn. Find the probability that "the first and the second draws are red and the third one is green".

4. Four fair coins are flipped. If the outcomes are assumed independent, what is the probability that two heads and two tails are obtained?

5. We have six guns of three possible types with labels A, B, and C. The guns have the same look. The probability of scoring the target with gun A, B, C is 0.5, 0.7, 0.8, respectively. Further, we have 3 guns of type A, 2 guns of type B, and 1 gun of type C. We randomly select a gun, find the probability that we score the target. What is the conditional probability that we use gun C given that we score the target?

6. In a certain city, 30 percent of the people are Conservatives, 50 percent are Liberals, and 20 percent are Independents. Records show that in a particular election 65 percent of the Conservatives voted, 82 percent of the Liberals voted, and 50 percent of the Independents voted. If a person in the city is selected at random and it is learnt that he did not vote in the last election, what is the probability that he is a Liberal?

7. In ping-pong the first player who achieves 11 points is the winner of the match. However, the winner should win with a difference of at least 2 points. Hence if the current state is 10-10 then the game continues until one of the players has an advantage of 2. In a championship the champion win 100000 HUF. The final match of this championship is played by two equally strong players. During this final match there is a powerbreak. Before the powerbreak the state of the game was 10-9. It is not possible to finish the game after the powerbreak. Hence, it is decided to divide the price money among the two finalists. In which proportion it would be fair to divide the price money?

8. In a certain country, 60 percent of the population is right-handed, 40 percent is left-handed. A right-handed person is able to hit a certain target with his left hand with a probability 0.2. For a left-handed person this probability is de?nitely larger, it is 0.7, A person is chosen at random.

   a) What is the probability that a person hits the target with his/her left hand?

   b) Assume that you get the information that a person hit the target with the left hand. What is the probability that that person is left-handed?

9. We roll two fair dice a blue and a red. We first roll the red die then we roll the blue die as many times as the outcome of the red die. Let Y denote the outcome of the red die and denote by X the sum of the outcomes on the blue die.

   a) Find $E(X)$ and $Var(X)$.

b) What is the sign of $cov(X, Y)$?

10. A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that will take him to safety after 3 hours of travel. The second door leads to a tunnel that will return him to the mine after 5 hours of travel. The third door leads to a tunnel that will return him to the mine after 7 hours. Assume that the miner is at all times equally likely to choose any one of the doors. Find the expected length of time until he reaches safety.