Midterm exam 1

24 October 2012. 18:00

Advanced Mathematics for Electrical Engineers B, Stochastics part

Working time: 100 minutes. Every exercise is worth 9 points.

1. A certain kind of plant lives for exactly one year. Before dying, it leaves a random number of offspring, independently of the past and of other members of the population. The number of offspring can be 0, 1, 2 or 3 with equal probabilities. At time 0 (in the zero-th year) there is a single plant in a population.

   Let Z_k denote the size of the k-th generation ($k = 0, 1, 2, \dots$) (that is, the number of plants in the k-th year. Model the system by a dicrete time branching process.

   a.) What is the generating function of Z_2?

   b.) What is the expectation of Z_{10}?

   c.) How much is the probability $\mathbb{P}(Z_3 = 0)$?

   d.) What is the probability that the population dies out (that is, one of the generations will already be empty)?

2. On an airplane, 300 passengers will travel, whose weights are random and independent. The expectation of the total weight of all passengers is known to be 21000 kg. Every passenger is at least 10 kg heavy (no small babies allowed), and none of them can weigh more that 150 kg (would not fit into his seat).

   How much weight should the airplane be able to lift (counting passengers only), if we want to be $1 - 10^{-8}$ sure that the total weight of passengers doesn't exceed that? Give a usable bound!

3. A child is jumping up and down the 3 stairs at home. So, counting from the ground, she can be 0, 1, 2 or 3 stairs up. At every "jump", she jumps up one step with probability 2/3 unless she is already at the top, and jumps down one step with probability 1/3 unless she is already at the bottom, independently of the past. If she is on top, she stays (jumps where she was) with probability 2/3, and jumps down one step with probability 1/3, while if at the bottom, she jumps up one step with probability 2/3 and stays at the bottom with probability 1/3. Suppose she starts from the bottom. Model her position with a Markov chain.

   a.) Draw the transition graph of the Markov chain.

   b.) Give the transition probability matrix of the Markov chain.

   c.) What is the probability that she's 2 stairs up after 4 jumps?

   d.) After 100 jumps, what is the approximate probability that she's at the top?

e.) What is her average height (measured in stairs) on the long run?

4. We took a sample from a random variable which is known to have (optimistic) geometrical distribution with an unknown parameter p. The result of the sampling is $4, 12, 7, 1, 3, 1, 1, 2, 3, 3, 6$. Give a maximum likelyhood estimate for the parameter p.

5. A random variable is normally distributed with unknown mean μ and unknown variance σ^2, but we have the *hypothesis* that $\mu = 900$. In order to test our hypothesis, we took a sample with $n = 10$ elements, and got the numbers $901.9\,,\ 906.4\,,\ 894.9\,,\ 897.7\,,\ 899.1\,,\ 900.9\,,\ 899.9\,,\ 903.8\,,\ 908.1\,,\ 904.8\,.$ Decide, at a confidence level of 95%, whether the hypothesis is true or not. (*Help: for the above sequence of numbers, the sum* $\sum_i x_i$ *is* 9017.5*, while the sum of squares* $\sum_i x_i^2$ *is* 8131680.59*. From these,*

$$\frac{1}{n}\sum_i x_i^2 - \left(\frac{1}{n}\sum_i x_i\right)^2 = 14.9965$$

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