

# Stochastics exam

13 January 2015. 13:00

Advanced Mathematics for Electrical Engineers B

Working time:  $\leq 60$  minutes.

1. (8 points) Let  $Z_n$  be a Galton-Watson branching process, in which  $Z_0 = 1$ . What is the probability that the process dies out, if the one-step offspring distribution is

a.) (2 points) 
$$\frac{k}{\mathbb{P}(k \text{ children})} \quad \left| \begin{array}{c|c|c} 0 & 1 & 2 \\ \hline 1/4 & 1/4 & 1/2 \end{array} \right.$$

b.) (2 points) 
$$\frac{k}{\mathbb{P}(k \text{ children})} \quad \left| \begin{array}{c|c|c|c|c|c} 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 7/10 & 1/10 & 0 & 1/10 & 0 & 1/10 \end{array} \right.$$

c.) (2 points) Poisson with parameter  $\lambda = 1$ ,

d.) (2 points) such that the number of children is always 1.

2. (7 points) On a written exam 200 students take part. Each of them can reach at most 100 points, and at least 0. The scores reached by the students are independent, with expectation 50. Give a large deviation estimate for the probability that the average score is at most 40.

3. (10 points) In a country there are 4 classes of the football league. The “Uni Varsity Football Club” starts (in year 0) in the 4-th league. From the 4th league, they move after a year to the 3rd league with probability  $\frac{1}{3}$ . If they are in the 3rd league, they move after a year to the 2nd with probability  $\frac{1}{5}$ , and to the 4th with probability  $\frac{1}{6}$ . If they are in the 2nd league, they move to the 1st with probability  $\frac{1}{8}$ , and to the 3rd with probability  $\frac{1}{5}$ . Finally, if they are in the 1st league, they move to the 2nd with probability  $\frac{1}{4}$ . With the remaining probabilities they stay in the league where they were. The moves of each year are independent of what happened before.

Let  $X_n$  denote the number of the league in which they play after  $n$  years.

a.) (1 point) Draw the graph representation of the Markov chain  $X_n$ .

b.) (2 points) Give the state space and the transition matrix of the Markov chain.

c.) (3 points) Find the stationary distributions of the Markov chain.

d.) (2 points) What is the approximate probability that the team will play in the 4th league after 30 years? Why?

e.) (2 points) In the 4th league they have 500 spectators a year, in the 3rd they have 1000, in the 2nd they have 2000, and in the 1st they have 4000. What will be the average number of spectators per year on the long run? Why?