Stochastics exam

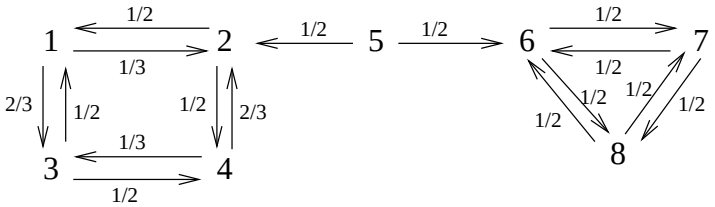
20 January 2015. 13:00

Advanced Mathematics for Electrical Engineers B

Working time: $\leq$60 minutes.

1. *(7 points)* The result of an experiment can be one of the numbers $0, 1, 2, 3, 4, 5$, with equal probability. We repeat the experiment 500000 times independently. Give a large deviation estimate for the probability that the average of the results reaches 5.2.

2. *(8 points)* Let the discrete time Markov chain X_n have the following graph representation:



Give the approximate value of the probabilities below. Justify your answer.

a.) *(2 points)* $\mathbb{P}(X_{1000} = 7 \,|\, X_0 = 6) \approx ?$

b.) *(2 points)* $\mathbb{P}(X_{1000} = 2 \,|\, X_0 = 1) \approx ?$

c.) *(2 points)* $\mathbb{P}(X_{1000} = 2 \,|\, X_0 = 6) \approx ?$

d.) *(2 points)* $\mathbb{P}(X_{1000} = 7 \,|\, X_0 = 5) \approx ?$

3. *(10 points)* Anne has three favourite dresses: 1: a red; 2: a blue; 3: a yellow one. Every day she wears one of them. She never wears the same dress on two consecutive days. If she wears the red one day, then the next day she is sure to wear the blue one. Similarly, after the blue one she is sure to wear the yellow. However, after the yellow dress, she decides about the next day (red or blue) by coin tossing. Let X_n denote the number of the dress she is wearing on the nth day of the year.

a.) *(1 point)* Draw the transition graph of the Markov chain X_n.

b.) *(3 points)* Give the transition probability matrix of the Markov chain X_n.

c.) *(3 points)* Find the stationary distributions of the Markov chain.

d.) *(1 point)* Assume that she is wearing her yellow dress on day 0 (31 December). What is the probability that she is wearing it again on day 3? Why?

e.) *(1 point)* Assume that she is wearing her yellow dress on day 0 (31 December). What is the approximate probability that she is wearing it again on day 300? Why?

f.) *(1 point)* On what percentage of the days will she wear her yellow dress on the long run? Why?