Stochastics exam

27 January 2015. 13:00

Advanced Mathematics for Electrical Engineers B

Working time: $\leq$60 minutes.

1. *(7 points)* In a probabilistic computer algorithm every process starts a random number of "child processes" before it quits. This random number of child processes is independent of the past of the algorithm, and has the following distribution:

k	0	1	2
$\mathbb{P}(k \text{ children})$	1/3	1/3	1/3

. Initially, there is a single process, called the "root process". The algorithm completes succesfully when every process quits.

   a.) *(2 points)* What is the probability that the children of the root process have no children? (That is, the 2nd generation of processes is already empty?)

   b.) *(3 points)* What is the probability that the algorithm ever completes succesfully?

   c.) *(2 points)* Every process takes 1 millisecond of processor time. What is the expectation of the total time it takes for the algorithm to complete?

2. *(8 points)* In an electric network there are 10000 consumers. 9000 of them use $32A$ fuses, so their power consumption can be no more than $32A \times 230V = 7360W$. The other 1000 consumers have $100A$ fuses, so their power consumption can be at most $100A \times 230V = 23000W$. According to the experience of the network operator, (in peak hours) the *average total* consumption of the consumers is $3.2 \cdot 10^7 W$. Give a large deviation estimate for the probability that the total consumption (at some given peak time) is at least $3.5 \cdot 10^7 W$.

3. *(10 points)* Let X_n $(n = 1, 2, \dots)$ be a discrete time Markov chain on the state space $S = \{1, 2, 3\}$ with the transition probability matrix

$$P = \begin{pmatrix} 1/2 & 1/3 & 1/6 \\ 1/6 & 1/2 & 1/3 \\ 1/3 & 1/6 & 1/2 \end{pmatrix}.$$

   a.) *(1 point)* Draw the graph representation of the Markov chain.

   b.) *(2 points)* Calculate the conditional probability $\mathbb{P}(X_2 = 3 \,|\, X_0 = 1)$.

   c.) *(3 points)* Find the stationary distributions of the Markov chain.

   d.) *(2 points)* What is the approximate value of the probabiilty $\mathbb{P}(X_{50} = 3 \,|\, X_0 = 1)$? Why?

   e.) *(2 points)* Let the observable $f : S \to \mathbf{R}$ be given by $f(1) = 1$, $f(2) = 4$, $f(3) = 9$. What is the limit (as $N \to \infty$) of the time average

$$\frac{f(X_0) + f(X_1) + \cdots + f(X_{N-1})}{N},$$

   and why?