2-nd resit of midterm exam 1

19 December 2014. 10:00

Advanced Mathematics for Electrical Engineers B, Stochastics part

Working time: 70 minutes. Every exercise is worth 7 points.

1. A certain kind of plant lives for exactly one year. Before dying, it leaves a random number X of offspring, independently of the past and of other members of the population. The distribution of the random variable X is

k	0	1	2	3
$\mathbb{P}(X = k)$	1/4	1/4	1/4	1/4

   At time 0 (in the zero-th year) there is a single plant in a population.

   Let Z_k denote the size of the k-th generation ($k = 0, 1, 2, \dots$) (that is, the number of plants in the k-th year). Model the system by a dicrete time branching process.

   a.) What is the generating function of Z_2?

   b.) What is the expectation of Z_{10}?

   c.) How much is the probability $\mathbb{P}(Z_3 = 0)$?

   d.) What is the probability that the population dies out (that is, one of the generations will already be empty)? *(Hint: $z^3 + z^2 - 3z + 1 = (z-1)(z^2 + 2z - 1)$.)*

2. On an airplane, 300 passengers will travel, whose weights are random and independent. The expectation of the total weight of all passengers is known to be 21000 kg. The weight of each passenger is at least 10 kg and at most 150 kg.

   Let K be the maximum weight (of passengers) that the airplane can lift (measured in kilograms), How much should K be, if we want to be $1 - 10^{-8}$ sure that the total weight of passengers doesn't exceed that? Give a usable bound!

3. Let X_n be a Markov chain of the state space $S = \{1, 2, 3, 4\}$ with the following rule:

   - with probability $\frac{2}{3}$ the system jumps 1 step "up" (unless it is already at 4: if it is at 4, it stays there).
   - with probability $\frac{1}{3}$ the system jumps 1 step "down" (unless it is already at 1: if it is at 1, it stays there).

   a.) Draw the transition graph of the Markov chain.

   b.) Give the transition probability matrix of the Markov chain.

   c.) What is the conditional probability $\mathbb{P}(X_4 = 3 \mid X_0 = 1)$?

   d.) What is the approximate probability of $\{X_{100} = 4\}$?

   e.) What is the average of X_n on the long run?