Resit of midterm exam 1

12 December 2014. 10:00

Advanced Mathematics for Electrical Engineers B, Stochastics part

Working time: 70 minutes. Every exercise is worth 7 points.

1. We keep rolling a fair die until we first roll a 6. Let S denote the sum of the numbers rolled *before* (and not including) that 6. Calculate the generating function and expectation of S.

   (Hint:

   *a.) Let X be the result of a fair die roll which is known **not to be 6**. So – knowing this – what is the distribution of X, and what is its generating function?)*

   b.) Let N be the number of rolls by a fair die until (but not including) the first 6. What is the distribution of N and what is its generating function?)

2. In a certain village there is a person called Harry WhoIam. There is no other person with the family name "WhoIam". This village is famous for its strict birth regulations: each person has exactly 2 children. On the other hand, there is a tendency for more boys than girls to be born: the probability that the sex of a newborn is male is exactly $\frac{3}{5}$ (independently of the other births).

   The village follows the conventional naming pattern: every child inherits the family name from the father. (So, in the story of the WhoIam family name, only boys play a role.)

   a.) Let X denote the number of sons (male children) of Harry WhoIam. What is the distribution of X?

   b.) What is the probability that Harry WhoIam will have at least one grandson (male grandchild) with the same family name WhoIam?

   c.) What is the probability that the family name WhoIam dies out in the village?

3. The planned East-Siberia oil pipeline will carry the production of 700 oil wells. The daily production of these oil wells is random and independent, the expectation of their total daily production is 560000 barrels. It is also known that 400 of the wells always have their daily production between 880 and 1380 barrels, while the other 300 wells always produce between 200 and 1200 barrels daily. What should the daily carrying capacity of the pipeline be, if we want the probability of an overload (larger daily total production than daily carrying capacity) to be at most 10^{-4}? (Give a bound which is better than the total maximum production of the wells.)