Exam sheet

Mathematics, Part 1: Probability Theory and Applications

Final Exam

December 20, 2011

Time: 70 minutes

1. (3 points) Define the notion of consistent estimator. Show an example of a consistent estimator.

2. (6 points) The daily energy consumption of a town has normal distribution with unknown standard deviation σ and known mean μ. We measured the energy consumption of the city on n days. The results are: $x_1, \ldots, x_n$. Based on this sample find the Maximum Likelyhood estimator of σ.

3. (5 points) A town is inhebited by 40000 families. The amount of garbage produced by a family in a day is no more than 50 liters, the expectation is 20 liters. The town installs a trash-burning plant. Find a capacity K for the plant such that the total amount of garbage produced by all the families in a day, can exceed the capacity K with a probability at most 10^{-6}.

4. (11 points) Passengers arrive at a train station. The waiting room has a finite capacity of 4 passengers. Let $X(t)$ be the number of passengers in the room at time t. Assume that $X(t), t \geq 0$ is a five state continuous time Markov chain with state space $S = \{0, 1, 2, 3, 4\}$ and transition probability matrix

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   Assume that if the number of passengers waiting is 0, 1, 2, 3 and 4, then the waiting time until the next jump has distribution $Exp(1), Exp(4), Exp(4), Exp(4) and Exp(2)$, respectively.

   (a) Find the graph representation of the Markov chain.

   (b) Find the percentage of time (on the long run) that the number of passengers waiting is j, for $j = 0, 1, 2, 3, 4$.

   (c) The cost rate of staying in state i is $i + 2$, $i = 0, 1, 2, \ldots, 4$. Find the long-run average cost of the maintenance of the waiting room.