Exam sheet

Mathematics, Part 1: Probability Theory and Applications

Final Exam

January 3, 2012

Time: 70 minutes

1. (3 points) Define the notion of consistent estimation. Show an example of a consistent estimator.

2. (6 points) In 2015 a local internet service provider serves 12000 users. In the peak hours. based on their subscriptions and behaviors, the users fall into one of the following three categories:

   - beginner: the bandwidth consumption is 100 Mbps in average but no more than 200 Mbps;
   - advanced: the bandwidth consumption is 160 Mbps in average but no more than 280 Mbps;
   - power user: the bandwidth consumption is 250 Mbps in average but no more than 400 Mbps;

   In these groups there are 3500, 6500 and 2000 users, respectively. Find a minimal bandwidth capacity C such that the probability that the capacity C is not enough is certainly less than 10^{-6}.

3. (7 points) Let $\vec{x} = (8, 10, 10, 8, 5, 7, 9, 8, 8, 8)$ be a realization of the i.i.d. sample $X_1, \ldots, X_{10}$ from the $Binomial(20, p)$ distribution with parameter p, where $p \in (0, 1)$. Find the maximum likelihood estimation of p.

4. (9 points) *On-Off System.* Anne and Bob live together. Anne likes warm, while Bob likes cold. Whenever the thermostat in their house is turned up, Bob will pass by within some random time with exponential distribution with mean $1/\lambda$, and turn it down. When the thermostat is turned down, Anne will come in some random time with exponential distribution with mean $1/\mu$, and turn it up. Let $X(t)$ be the state of the thermostat at time t. Suppose the state space is $\mathcal{S} = \{0, 1\}$ with 0 corresponding to "down" and 1 corresponding to "up". Find the percentage of time in which the thermostat is turned up.