Stochastics exam

14 January 2014. 12:00

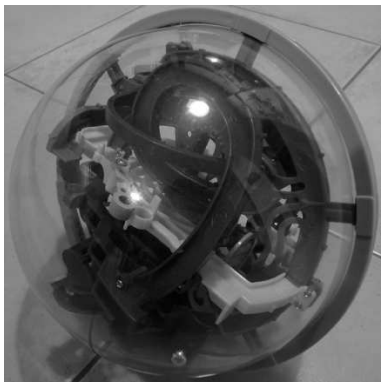
Advanced Mathematics for Electrical Engineers B

Working time: 60 minutes.

1. Let Z_k be a Galton-Watson branching process, where the one-step offspring distribution has generating function $g(z) = e^{z-1}$. What is the probability that sooner or later the process dies out? *(5 points)*

2. On an exam 120 students show up, 90 of whom are prepared, and the other 30 are unprepared. Those who are prepared, pass the exam with 90% probability, while those who are unprepared have only 30% probability of passing. Give a large deviation estimate for the probability that at least half of the students fail. *(6 points)*

3. Bill is playing a game of skill, where he has to drive a ball along a tricky track. He is practicing the first track, where there are 3 difficult obstacles. On the first obstacle he *fails* with probability $\frac{1}{4}$; on the second with probability $\frac{1}{3}$, and on the third with probability $\frac{1}{2}$, independently of what happened before. When that happens, the ball "falls down", and he can start all over again. In the opposite case he passes to the next obstacle. If, by chance, he manages to pass all three obstacles, then he also starts over from the beginning. Let X_n denote how many obstacles he has just passed after n steps – so the possible values of X_n are $0, 1, 2, 3$.



Magical Intellect Ball

   a.) Give the transition probability matrix of the Markov chain X_n. *(2 points)*

   b.) In which state will the Markov chain be most often on the long run, and what portion of the steps will Bill spend with this most met obstacle? *(4 points)*

   c.) On which obstacle will Bill *fail* most often on the long run, and what portion of his failures will happen on this obstacle? *(2 points)*

4. On a (possibly) biased die the probability of rolling 6 is some unknown $p \in (0; 1)$, while all the other numbers have the same probability $\frac{1-p}{5}$. We rolled this die 10 times to obtain a sample form the distribution, and we got $5; 6; 4; 3; 4; 6; 3; 1; 6; 3$. Give a maximum likelihood estimate for the value of p. *(6 points)*