Midterm exam 1 resit

12 November 2012. 18:00

Advanced Mathematics for Electrical Engineers B, Stochastics part

Working time: 100 minutes. Every exercise is worth 9 points.

1. In a physical experiment the researchers are trying to observe radiactive decay events. In their sample, the number of decays that occur is random, and has a Poisson distribution with parameter 30. A decay event that *occurs* may or may not be *observed* by the instruments: each of them is *observed* with probability $\frac{1}{3}$, independently of the others. What is the distribution of the number of decays *that are observed*?

   (Hint: a possible solution is to calculate the generating function.)

2. A truck is carrying 1000 small, 2000 medium size and 1000 big parcels. A small parcel can weigh at most 1 kg, a medium size parcel at most 2 kg, and a big one at most 5 kg. However, the actual weights are random (and independent of each other), with the expectations being only 0.5 kg, 1 kg and 3 kg for the small, medium and big parcels, respectively. The truck is allowed to carry 6000 kg. Estimate the probability that it is overloaded.

3. Figure 1 shows the graph representation of a dicrete time Markov chain with 3 states, indicating the transition probabilities.

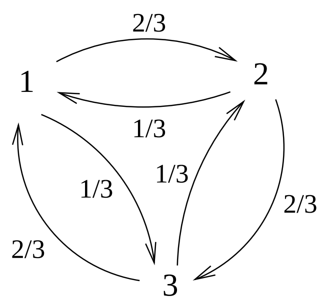

Figure 1: Graph representation of a Markov chain

   We start from state "1".

   a.) What is the probability of observing the sequence "12323213" as the initial sequence of states?

   b.) What is the probability of being in state "1" again after 3 steps?

   c.) Give the Markov transition matrix.

   d.) Calculate the approximate probability of being in state "2" after 100 steps.

4. Anne and Bob live together. Anne likes warm, so at random times she turns on the heating (if it's not already on). Bob likes cold, so at random times he turns off the heating (if it's not already off). These events of turning "on" occur at exponentially distributed time intervals with parameter 1, while the events of turning "off" occur at exponentially distributed time intervals with parameter 1/2, independently of each other and the past.

a.) Model the state of the heating switch with a continuous time Markov chain. What are the states?

b.) Give the jump rates from each state and the transition matrix of the embedded discrete time Markov chain.

c.) Give the infinitesimal generator.

d.) After a long time, what is the approximate probability that the heating is turned on?

e.) On the long run, in what proportion of the time will the heating be turned on?

5. We took a sample from a random variable which is known to have exponential distribution with an unknown parameter λ. The result of the sampling is 16.3, 12.1, 11.5, 1.8, 50.7, 18.3, 15.7, 1.8, 10.2, 11.5. Give a maximum likelyhood estimate for the parameter λ.