# Problem Sheet - Third practical course

1. Find the generator functions of the pessimistic and the optimistic geometric distributions!

2. In a certain village there is a person with name Harry WhoIam. There is no other person with this last name. This village is famous from the fact that each person have 3 children. The sex of the children is equally likely to be male or female independently of each other. The newborns get the last name of their fathers.

   a) What is the probability that Harry WhoIam will have grandson with last name WhoIam?

   b) What is the expected number of grandsons with last name WhoIam?

   c) What is the probability of the extinction of last name WhoIam in the village?

3. Assume that a maleware is infecting the computers of an infinite size population of computers. If a computer is infected then by the end of the next day one of the following possibilities occurs. It is detected and deleted with probability p. Consequently, it is not infectious anymore. With probability $(1 - p)p$ it is not deleted and does not infect other computer. Further, it is not deleted and it infects $k - 1$ new computers having not infected to that date with probability $(1 - p)p^k$ for $k = 2, 3, \ldots$. Assume that the contribution of each infected computer is independent of the other infected computers' contributions. In the end of 0'th day there is one infected computer. Answer the following questions in case of $p = \frac{1}{4}$ and in case of $p = \frac{2}{3}$.

   a) Find the expected number of infected computers in the end of 30'th day!

   b) What is the probability that there is not infected computer in the end of the 3'th day?

   c) What is the probability that the maleware will be never perished?

4. In order to help some friends, Harry becomes the east cost sales representative of B& D Software. The software has been favorably reviewed and demand is heavy. Harry sets up a sales booth at the local computer show and takes orders. Each order takes three minutes to fill. While each order is being filled there is a probability $p_j$ that $j$ more customers will arrive and join the line. Assume $p_0 = \frac{1}{2}$, $p_1 = \frac{1}{3}$ and $p_2 = \frac{1}{6}$. Harry cannot take a coffee break until a service is completed and no one is waiting in the line.

   a) What is the probability that Harry will have coffee break?

   b) What is the expected number of customers he serves before the first coffee break?

   c) What is the generator function of the number of customers he serves before the first coffee break?

   d) Answer the previous questions if $p_0 = \frac{1}{6}$, $p_1 = \frac{1}{3}$ and $p_2 = \frac{3}{6}$!

5. Let $\alpha(p)$ denotes the probability of non-extinction of a branching process of which offspring distribution is pessimistic geometric with parameter $p$. Draw the $p \to \alpha(p)$ function!