Problem Sheet # 3

Large Deviations

September 21, 2011

1) In a cross the average level of the noise pollution is 45dB. We measure the noise level 100 times. We have observed that the level of the noise is larger than 50 dB 10 times. Find the frequency of measuring noise level smaller than 37dB.

2) IQ is normally distributed with a mean of 100 and a standard deviation of 15. Suppose one individual is randomly chosen. Find the probability that the person has an IQ greater than 120.

3) In 2017 a local internet service provider serves 12000 users. In the peak hours based on their subscriptions and behaviors the users fall into one of the following three categories:

   - beginner: the bandwidth consumption is 100 Mbps in average but no more than 200 Mbps;
   - advanced: the bandwidth consumption is 160 Mbps in average but no more than 280 Mbps;
   - power user: the bandwidth consumption is 250 Mbps in average but no more than 400 Mbps;

   In these groups there are 3500, 6500, 2000 users respectively. Find the minimal bandwidth capacity C such that the probability that the capacity C is not enough is less than 10^{-6}.

4) An entrepreneur owns a company in the building industry. At the beginning of each months the company signs a new contract with probability 70%. The work is always the same and it takes one month. The income is 10,000€. If no contract is signed then there is no income.
   a) Find the expected value of the income in 30 months.
   b) Estimate the likelihood that the income is smaller than 200,000€ in 30 months.
   c) Estimate the likelihood that the income is smaller than 150,000€ in 30 months.

5) All around a big country wind power plants will be installed. The capacity of turbine type A is between 0.5 MW and 1.6 MW the average capacity is 1 MW. The capacity of turbine type B is between 1.2 MW and 2.8 MW the average capacity is 2 MW. Find the aggregated capacity that is not exceeded with probability 10^{-6} if there will be installed 400 turbines of type A and 200 turbines of type B.

6) In a town 40000 families live. The amount of garbage produced by a family in a day is no more than 50 liters, the expectation is 20 liters and the standard deviation is 10 liters.
   a) The town installs a trash-burning plant. Find the capacity of the plant such that the amount of garbage per day is less than the capacity with probability at least 1%. Apply CLT to estimate the capacity.
   b) The government of the town modifies the probability of fault to 10^{-8}. Find the necessary capacity of the plant by using Hoeffding inequality.

7) Harry is playing roulette in a casino. In each game he stakes 1000€ in the outcome of "red". By the end of the 100th game his loss is 3000€. Shoud he suspect that the casino is cheating?