Problem Sheet # 1

September 7, 2011

1) A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy? Assume that the sample space S is given by $S = [(b, b), (b, g), (g, b), (g, g)]$, and all outcomes are equally likely. [(b, g) means for instance that the older child is a boy and the younger child a girl.]

2) Suppose we toss two fair dice. Let E_1 denote the event that the sum of the dice is six and F denote the event that the first die equals four. Show that E_1 and F are not independent. Let E_2 be the event that the sum of the dice equals seven. Is E_2 independent of F?

3) Player A and player B toss a coin having a probability p $(0 < p < 1)$ of coming up heads, until the first head appears. The order of the throws are $A, B, A, \ldots$. What is the probability that A wins.

4) Four fair coins are flipped. If the outcomes are assumed independent, what is the probability that two heads and two tails are obtained?

5) An exam consists of 20 questions. For each question there are 2 possible answers Yes or No. Regarding the student filling the test there are three possibilities. The answer is right with probability $4/7$. The answer is wrong (the student thinks that it is right) with probability $2/7$. The student does not know the answer with probability $1/7$. In this case the student tosses a fair coin if it is head then the answer is Yes, if the coin is tail then the answer is No. What is the probability of the right answer?

6) We have six guns of three possible types with labels A, B, and C. The guns have the same look. The probability of scoring the target with gun A, B, C is 0.5, 0.7, 0.8, respectively. Further, we have 3 guns of type A, 2 guns of type B, and 1 gun of type C. We randomly select a gun, find the probability that we score the target.

7) A social network over the Internet is growing only by invitations. It starts with two individuals Adam and Eve. On each day a (uniformly) randomly selected member of the community invites an individual from outside of the network. A community member is Adam‘s friend if he is Adam or he is invited by a member who is Adam‘s friend. If the community counts 4 people then find the probability that the number of Adam’s friends is 1, 2, and 3.

8) A simple component of some tool is made by mass production. The average number of bad items per day is 11. Find the probability that at most 20 bad items are produced in a day.

9) Suppose that the length of a telephone call (in minutes) is exponentially distributed with rate parameter $r = 0.2$. Find the probability that the call lasts between 2 and 7 minutes.

10) Suppose that the lifetime X of a fuse (in 100 hour units) is exponentially distributed with $\mathbf{P}(X > 10) = 0.8$. Find the rate parameter. Find the mean and standard deviation.

11) The position X of the first defect on a digital tape (in cm) has the exponential distribution with mean 100. Find the rate parameter. Find the probability that $X < 200$ given $X > 150$.

12) A book of 500 pages has 500 misprints. Using the Poisson distribution, estimate to three decimal places the probabilities that a given page contains (i) exactly 3 misprints, (ii) more than 3 misprints.

13) Suppose that requests to a web server follow the Poisson model with rate $r = 5.$ per minute. Find the probability that there will be at least 8 requests in a 2 minute period.

14) Defects in a certain type of wire follow the Poisson model with rate 1.5 per meter. Find the probability that there will be no more than 4 defects in a 2 meter piece of the wire.

15) Suppose that customers arrive at a service station according to the Poisson model, at a rate of $r = 4$. Find the mean and standard deviation of the number of customers in an 8 hour period.

16) In a cross the average level of the noise pollution is 45dB. We measure the noise level 100 times. We have observed that the level of the noise is larger than 50 dB 10 times. Find the frequency of measuring noise level smaller than 37dB.