# Quantum Information Theory and Mathematical Physics 2018

Budapest, 20-23.09.2018

# Program

	Thursday 20.09	Friday 21.09	Saturday 22.09	Sunday 23.09
9.30 - 10.30	Carlos Palazuelos	David Pérez García	Andreas Winter	Anna Jenčová
10.30 - 11.00	Coffee break			
11.00 - 12.00	Ludovico Lami	Marcus Huber	Tomohiro Ogawa	Péter Vrana
12.30 - 14.30	Lunch			
14.30 - 15.30	Guillaume Aubrun	Géza Tóth	Fumio Hiai	
15.30 - 16.00	Coffee break			
16.00 -	Free discussion, sightseeing, dinner			

# Conference dinner:

Time: Friday, 21 September, from 18.30 Location:

# Talks

# Speaker: Guillaume Aubrun

# Title: Universal gaps for XOR games from sharp estimates on tensor norm ratios II

**Abstract:** We present some new estimates on the gap between the injective and the projective tensor norms on a pair of finite-dimensional normed spaces, as a function of the dimensions. As we have seen in the talk by Ludovico, such gaps (and the related function r(n,m)) are closely related to the gap between local and global strategies in XOR games, in the context of general probabilistic theories.

No familiarity with the theory of tensor norms will be assumed ; we will first consider elementary examples, and then present some of the techniques we use to obtain lower bounds of r(n, m).

# Speaker: Fumio Hiai

# Title: Quantum *f*-divergences in operator algebras

Abstract: This talk surveys recent developments on quantum f-divergences in general von Neumann algebras and  $C^*$ -algebras, including standard f-divergences, maximal f-divergences and Rényi type divergences. Extending quantum f-divergences in the finite-dimensional matrix case, we introduce standard and maximal f-divergences in the von Neumann algebra setting, based on Haagerup's  $L^p$ -spaces and Araki's relative modular operators, and then present their basic properties such as joint convexity, joint lower semicontinuity, the monotonicity inequality under quantum operations, and martingale convergence, as well as their variational and integral expressions. In particular, we aim to characterize the equality case in the monotonicity inequality for quantum f-divergences toward the reversibility of quantum operations. We further present a general way how to extend quantum divergences in von Neumann algebras to those in  $C^*$ -algebras.

# Speaker: Marcus Huber

# Title: Encoding high-dimensional functions as tensor-edge hypergraphs

**Abstract:** Abstract: Boolean functions can be conveniently visualised via hypergraphs. Encoding these functions into the phases of quantum states then leads to the concept of hypergraph states, which appear in numerous quantum algorithms and constitute a highly interesting subclass of multipartite quantum states. For higher-dimensional functions over prime fields, hypergraphs are no longer sufficient to represent all possible functions. Using 'edge-tensors' we find an elegant representation of such functions that lead to the concept of tensor-edge hypergraph states, that have numerous interesting properties that we set out to explore.

#### Speaker: Anna Jenčová

#### Title: Incompatible measurements in general probabilistic theories

**Abstract:** We study incompatibility of measurements in quantum theory and in a broader class of finite dimensional general probabilistic theories. We introduce the notion of an incompatibility witness and present a geometric characterization of maximal incompatibility. Our approach allows to treat incompatibility, steering and non-locality in an unified manner and to observe that these non-classical effects are related to the geometry of polysimplices. As examples, we study the spaces of classical and quantum channels and show that the super-quantum non-classical effects observed on these spaces can be explained by their close relation to polysimplices. Details can be found in arxiv:1705.08008.

#### Speaker: Ludovico Lami

# Title: Universal gaps for XOR games from sharp estimates on tensor norm ratios I

Abstract: In this talk we will define and investigate XOR games in the framework of general probabilistic theories, which encompasses all physical models whose predictive power obeys minimal requirements. We will prove that the bias of an XOR game under local or global strategies is given by a certain injective or projective tensor norm, respectively. The intrinsic (i.e. model-independent) advantage of global over local strategies is thus connected to a universal function r(n,m) that we call the "projective-injective ratio". This is defined as the minimal constant of domination of the injective over the projective tensor norm across all products of the form  $X \otimes Y$ , where X and Y are Banach spaces of dimensions n and m, respectively. Our main results show that  $r(n,m) \ge c_1 \min\{n,m\}^{1/8}$  and  $r(n,n) \ge c_2 n^{1/6}$ for some universal constants  $c_1$  and  $c_2$  and for all positive integers n, m (more details in G. Aubrun's talk). The main operational consequence we draw here is that there is a universal gap between local and global strategies in general XOR games, and that this gap grows as a power of the minimal local dimension. In the quantum case, we are able to quantify it up to multiplicative constants. As a corollary, we obtain an improved bound on the scaling of the maximal quantum data hiding efficiency against local measurements.

#### Speaker: Tomohiro Ogawa

# Title: Information-spectrum approach for asymptotic convertibility of entanglement

**Abstract:** As an extension of previously known results on entanglement concentration and dilution, we consider asymptotic convertibility of an arbitrary sequence of bipartite pure states into another sequence by local operations and classical communication (LOCC). We adopt an information-spectrum approach to address cases where each element of the sequences is not necessarily a tensor power of a bipartite pure state. We derive a necessary condition and a sufficient condition [1] for the LOCC convertibility of one sequence to another, in terms of spectral entropy rates of entanglement of the sequences. As a byproduct, we show similar conditions for asymptotic convertibility of density operators by random unitary operations, which is applied to a study of quantum thermodynamics [2].

[1] Y. Jiao, E. Wakakuwa, T. Ogawa, J. Math Phys., vol 59, 022201, 2018.
[2] H. Tajima, E. Wakakuwa, T. Ogawa, arXiv:1611.06614, 2016.

#### Speaker: Carlos Palazuelos

#### Title: Classical XOR games with one-way communication

**Abstract:** Abstract: In this talk we will introduce classical XOR games and we will briefly review some classical results about their classical and entangled values. Then, we will consider the context where the players can use both classical and quantum communication as part of their strategies and we will explain some non-trivial bounds in this new scenario. Finally, we will explain some connections between this scenario and some problems in communication complexity.

As we will see, the study of XOR games is related to the theory of tensor norms in Banach spaces. This connection will link this talk to G. Aubrun's and L. Lami's talks.

#### Speaker: David Pérez García

#### **Title: Matrix Product Operators in Quantum Information**

**Abstract:** Matrix Product Operators were introduced by Fannes, Nachtergaele and Werner in their seminal 1992 paper. In this talk, I will show how their study can give new insights in different subjects within quantum information, such as channels capacities, quantum cellular automata or topological order in quantum spin systems.

#### Speaker: Géza Tóth

#### Title: Quantum Fisher information and entanglement

**Abstract:** First, we review important results, which, using the quantum Fisher information, show that separable quantum states outperform entangled ones in very general metrological tasks. In particular, the precision of the parameter estimation in a linear interferometer is bounded from below for separable quantum states. The bound is proportional to the particle number and is called the shot-noise limit. Entangled states can reach a much better precision, called the Heisenberg limit. In this case, the bound is proportional to the square of the particle number.

Then, we show that similar limits exist also for entangled states with a given entanglement depth. The larger the depth of entanglement, the better the quantum state for metrology. The best precision is reached for fully entangled states, for which the entanglement depth is maximal [1]. This makes it possible to detect the entanglement depth by precision measurements, which has already been used in experiments.

We also show that the quantum Fisher information can be defined as four times the convex roof of the variance [2]. This finding connects the quantum Fisher information to the theory of entanglement measures, which are defined often through convex roofs. Using this result, it is possible to estimate the quantum Fisher information based on some operator expectation values efficiently [3]. We also show a method to find a lower bound on the quantum Fisher information with the variance and the linear entropy of the state [4].

- [1] G. Tóth, Phys. Rev. A 85, 022322 (2012);
- P. Hyllus et al., Phys. Rev. A 85, 022321 (2012).
- [2] G. Tóth, D. Petz, Phys. Rev. A 87, 032324 (2013).
- [3] I. Apellaniz, M. Kleinmann, O. Gühne, G. Tóth, Phys. Rev. A 95, 032330 (2017).
- [4] G. Tóth, arXiv:1701.07461

### Speaker: Péter Vrana

### Title: Strassen's spectral theorem and relative submajorisation

**Abstract:** In his 1988 paper V. Strassen proved that given a semiring with a suitable preorder, the associated asymptotic preorder is characterised by monotone semiring homomorphisms into the nonnegative reals (spectral points). The main application of this "spectral theorem" was to tensors ordered by restriction or degeneration. Recently A. K. Jensen and I found that it also applies to the semiring of pure states ordered by trace-nonincreasing LOCC transformations, which leads to a characterisation of the trade-off between the asymptotic rate and the error exponent in the converse regime. It seems likely that the spectral theorem and yet-to-be-found generalisations thereof will become valuable tools for studying other resource theories as well. In my talk I describe a particular preordered semiring, determine its spectral points and explain some consequences of such a hypothetical generalisation.

#### **Speaker:** Andreas Winter

#### Title: Quantum Slepian-Wolf: An Unfinished Story

**Abstract:** The distributed compression of correlated sources, as well as its descendant problems of compression with side- information at the decoder, associated with the names of David Slepian and Jack Wolf in the classical setting, are a corner stone of multi-user Shannon theory. The attempts to generalise these fundamental tools to the quantum Shannon setting have met with mixed success, ranging from the deep (quantum state merging) to the disappointing (quantum data compression with classical side information). I will take up the latter problem and show some recent advances made in joint work with Zahra Khanian [to appear]. We show that, while for the classical case compression all the way down to the conditional entropy is possible, meaning that the rate sum always equals the Shannon limit, in the quantum case the minimum rate is generally strictly larger than the conditional entropy (of the quantum source conditioned on the classical side information). In other words, the rate sum is in general strictly larger than the Schumacher limit. We interpret this as the cost of ignorance of the encoder: If he had access to the side information as well as the decoder, the conditional entropy could be achieved, without it the compression rate is strictly larger.

# Posters

#### Presenter: Andreas Bluhm

#### Title: Joint measurability and the matrix jewel

Abstract: In this work, we investigate the joint measurability of quantum observables and connect it to the study of free spectrahedra. Free spectrahedra typically arise as matricial relaxations of linear matrix inequalities. Examples of free spectrahedra are the matrix diamond, which is a matricial relaxation of the  $l_1$ -ball, and its generalization the matrix jewel. We find that joint measurability of POVMs is equivalent to the inclusion of the matrix jewel into the free spectrahedron defined by the effects under study. This connection allows us to use results about inclusion constants from free spectrahedra to quantify the degree of incompatibility of quantum measurements. In particular, we completely characterize the case of binary POVMs in which the dimension is exponential in the number of measurements.

#### Presenter: Máté Farkas

#### Title: Self-testing mutually unbiased bases in the prepare-and-measure scenario

Mutually unbiased bases (MUBs) constitute the canonical example of incom-Abstract: patible quantum measurements. One standard application of MUBs is the task known as quantum random access code (QRAC), in which classical information is encoded in a quantum system, and later part of it is recovered by performing a quantum measurement. We analyse a specific class of QRACs, known as the  $2^d \rightarrow 1$  QRAC, in which two classical dits are encoded in a *d*-dimensional quantum system. It is known that among rank-1 projective measurements MUBs give the best performance. We show (for every d) that this cannot be improved by employing non-projective measurements. Moreover, we show that the optimal performance can only be achieved by measurements which are rank-1 projective and mutually unbiased. In other words, the  $2^d \rightarrow 1$  QRAC is a self-test for a pair of MUBs in the prepareand-measure scenario. To make the self-testing statement robust we propose new measures which characterise how well a pair of (not necessarily projective) measurements satisfies the MUB conditions and show how to estimate these measures from the observed performance. Similarly, we derive explicit bounds on operational quantities like the incompatibility robustness or the amount of uncertainty generated by the uncharacterised measurements. For low dimensions the robustness of our bounds is comparable to that of currently available technology, which makes them relevant for existing experiments. Lastly, our results provide essential support for a recently proposed method for solving the long-standing existence problem of MUBs. This is a joint work with Jdrzej Kaniewski.

### Presenter: Martin Plávala

## Title: Conditions for the compatibility of channels and their connection to steering and Bell nonlocality

**Abstract:** We formulate steering by channels and Bell nonlocality of channels as generalizations of the well-known concepts of steering by measurements and Bell nonlocality of measurements. The generalization does not follow the standard line of thinking stemming from the EPR paradox, but introduces steering and Bell nonlocality as entanglement-assisted incompatibility tests. The proposed definitions are, in the special case of measurements, the same as the standard definitions, but not all of the known results for measurements generalize to channels. For example we show that for quantum channels steering is not a necessary condition for Bell nonlocality.

## Presenter: Szilárd Szalay

## Title: On multipartite entanglement and multipartite correlations

**Abstract:** We briefly review the partial separability based classification of mixed states of multipartite quantum systems of arbitrary number of subsystems. We show how this structure boils down in the case when not entanglement but correlation is considered. As special cases, we consider the notions of k-separability and k-producibility (as well as their correlation versions), reveal how these are dual to each other, and discuss some consequences. We also give the corresponding multipartite correlation and entanglement monotones, being the natural generalizations of mutual information, entanglement entropy and entanglement of formation, showing the same lattice structure as the classification (multipartite monotonicity).

The contribution is based on the works [PhysRevA 92, 042329 (2015)], [SciRep 7, 2237 (2017)], [arXiv:1806.04392 [quant-ph]], and on results unpublished yet.