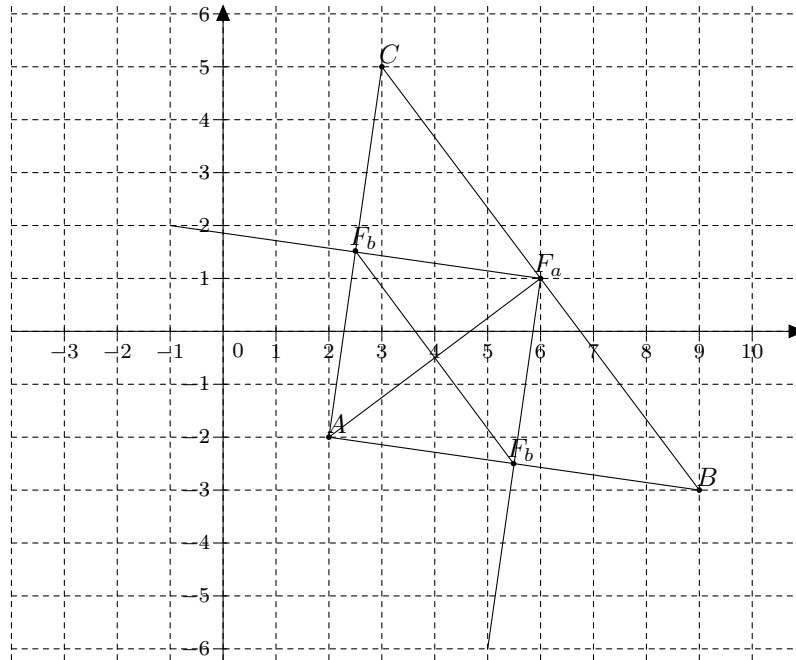


## Gyakorló feladatok

### 11. A, koordináta geometria 3. hét

1. Tekintsük az ábrán látható  $ABC$  egyenlőszárú háromszöget! Igazoljuk számítással, vagy vektorokkal, hogy derékszögű! Adjuk meg az oldalegyenesek egy-egy irányvektorát, az oldalak felezőpontjait, a középvonalak vektorait, az oldalfelező merőlegesek egy-egy irányvektorát!



Számítással:

Leolvasással:

$$\mathbf{v}_a = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_b = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_c = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$F_a = \left( \frac{\phantom{0} + \phantom{0}}{2}; \frac{\phantom{0} + \phantom{0}}{2} \right) = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$F_b = \left( \frac{\phantom{0} + \phantom{0}}{2}; \frac{\phantom{0} + \phantom{0}}{2} \right) = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$F_c = \left( \frac{\phantom{0} + \phantom{0}}{2}; \frac{\phantom{0} + \phantom{0}}{2} \right) = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{k_a} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{k_b} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{k_c} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{f_a} = \begin{pmatrix} -(\phantom{0}) \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

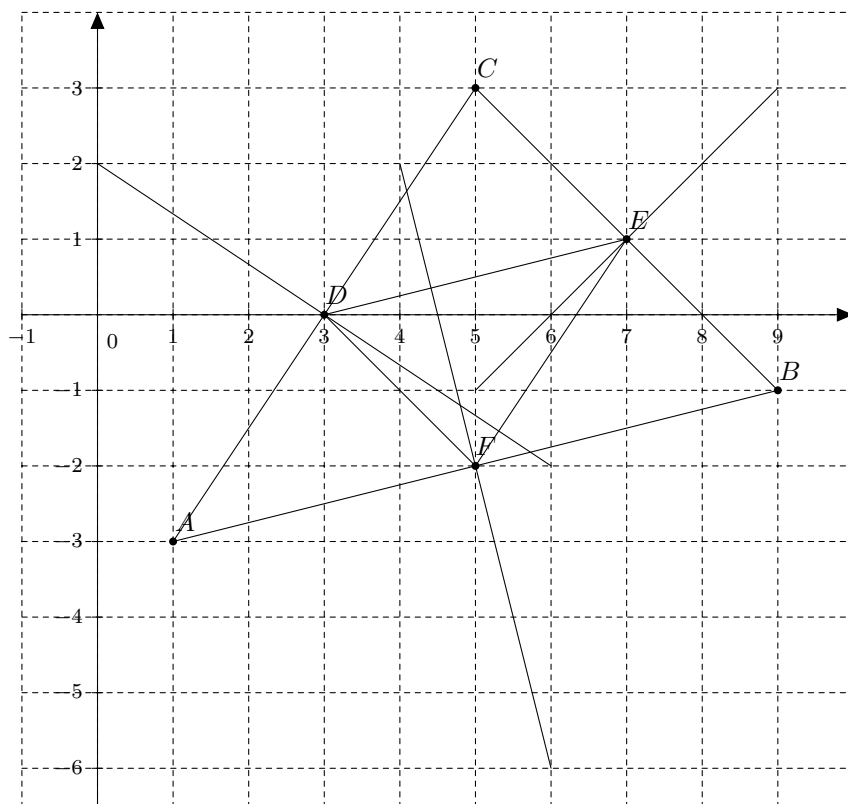
$$\mathbf{v}_{f_b} = \begin{pmatrix} -(\phantom{0}) \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{f_c} = \begin{pmatrix} -(\phantom{0}) \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

2. Tekintsük az ábrán látható  $ABC$  egyenlőszárú háromszöget! Adjuk meg az oldalegyenesek egy-egy irányvektorát, az oldalak felezőpontjait, a középvonalak vektorait, az oldalfelező merőlegesek egy-egy irányvektorát!



Számítással:

Leolvasással:

$$\mathbf{v}_a = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_b = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_c = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$F_a = \left( -\frac{\phantom{0}}{2}; -\frac{\phantom{0}}{2} \right) = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$F_b = \left( -\frac{\phantom{0}}{2}; -\frac{\phantom{0}}{2} \right) = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$F_c = \left( -\frac{\phantom{0}}{2}; -\frac{\phantom{0}}{2} \right) = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{k_a} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{k_b} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{k_c} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} - \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix} = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{f_a} = \left( -\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}; \phantom{0} \right) = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{f_b} = \left( -\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}; \phantom{0} \right) = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\mathbf{v}_{f_c} = \left( -\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}; \phantom{0} \right) = \begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$

$$\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$$