Komplex függvénytan

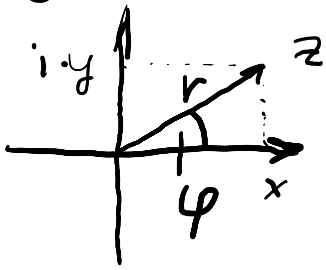
Komplex számok

$$a+bx \mid x^2 = x^2+1-1$$

$$\mathbb{C} = \{a+ib \mid a;b\in\mathbb{R}\} \cong \mathbb{R}[x]/(x^2+1) \cong$$

$$i^2=-1$$

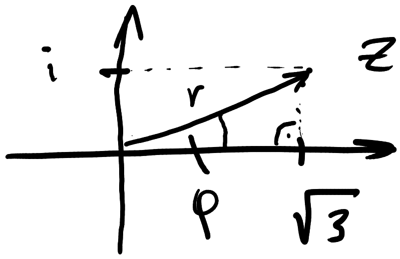
$$\cong \{M\in\mathbb{R}^{2\times 2} \mid M \text{ forgatva nyújtás}\}$$



$z = x+iy$

vld: $\mathrm{Re}(x+iy) = x \in \mathbb{R}$

kp: $\mathrm{Im}(x+iy) = y \in \mathbb{R}$



$$z = \sqrt{3} + i\cdot 1, \quad r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\mathrm{tg}\,\varphi = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \longrightarrow \varphi = \frac{\pi}{6}$$

- algebrai: $a+bi = r\cos\varphi + i\,r\cdot\sin\varphi$
- trigonometrikus: $r(\cos\varphi + i\sin\varphi)$
- exponenciális alak: $r\cdot e^{i\varphi}$

Euler formula
$$\boxed{e^{i\varphi} = \cos\varphi + i\sin\varphi} \qquad \varphi\in\mathbb{R}$$

$$z_1\cdot z_2 = r_1\cdot r_2(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2))$$

$$= r_1 e^{i\varphi_1}\cdot r_2 e^{i\varphi_2} = r_1\cdot r_2\cdot e^{i(\varphi_1+\varphi_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\cdot e^{i(\varphi_1-\varphi_2)} \quad ; \quad z^n = (r\cdot e^{i\varphi})^n = r^n\cdot e^{in\varphi}$$

$$\frac{1}{\sqrt{2}+i} = \frac{1}{\sqrt{2}+i}\cdot\frac{\sqrt{2}-i}{\sqrt{2}-i} = \frac{\sqrt{2}-i}{(\sqrt{2}+i)(\sqrt{2}-i)} =$$

$$= \frac{\sqrt{2}-i}{3} = \underbrace{\frac{\sqrt{2}}{3}}_{Re} - \underbrace{\frac{1}{3}}_{Im}\cdot i$$

$$(a+b)(a-b) = a^2-b^2 = 2-i^2 = 2-(-1) = 3$$

$$\overline{z} = \overline{a+bi} \overset{def.}{=} a-bi$$

konjugált

$$\overline{3+\sqrt{2}} = 3+\sqrt{2}$$

Komplex függvény

$$f: \mathbb{C} \supset\!\!\to \mathbb{C}\ ;\quad x+iy \mapsto f(x+iy)$$

$\mathrm{Dom}(f) \subseteq \mathbb{C}$

f valós és képzetes része $Re(f)$ $Im(f)$

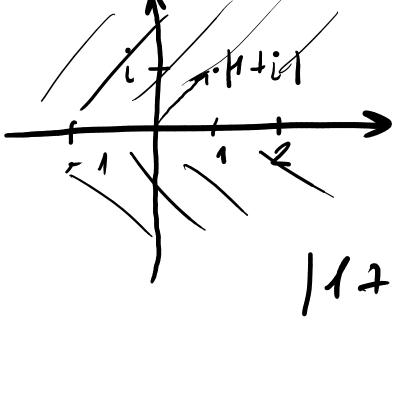
$$f(x+iy) = \underbrace{u(x,y)}_{\mathbb{R}^2\supset\to\mathbb{R}} + i\cdot\underbrace{v(x,y)}_{\mathbb{R}^2\supset\to\mathbb{R}}$$

1. $f(z) = z^2$ függvény mi a v. és k. része?

trükk $z = x + iy$, $x \in \mathbb{R}$, $y \in \mathbb{R}$ $\qquad i^2=-1$

$$(x+iy)^2 = x^2 + 2xiy + \underbrace{(iy)^2}_{-y^2} = \underbrace{x^2-y^2}_{u(x,y)} + i\cdot\underbrace{2xy}_{v(x,y)}$$

z



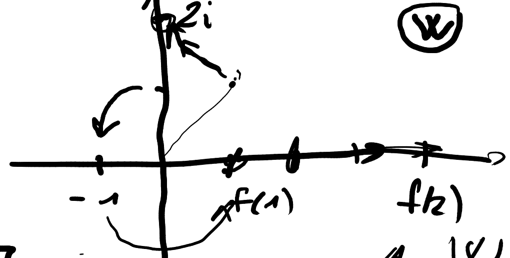
$(\ )^2$

$i^2=-1$

$(1+i)^2 =$
$= 1+2i+i^2 =$
$= 2i$

$$|1+i| = \sqrt{1^2+1^2}$$

w



$\mathbb{C}$

$\mathbb{C}$

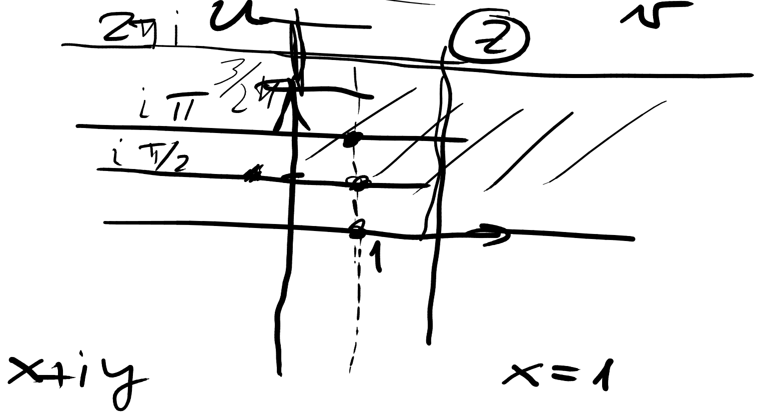
w_2

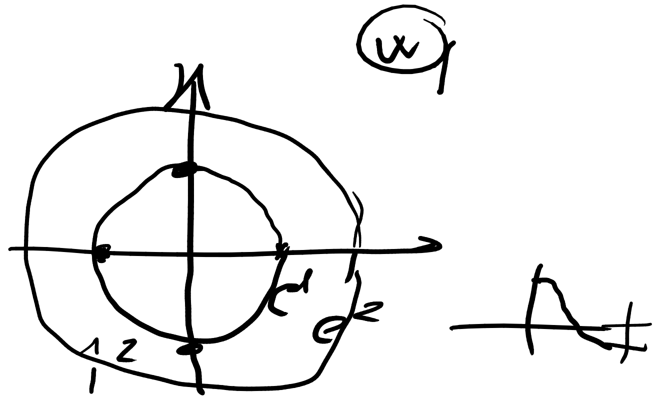
Exponenciális fv.

(2) $x+iy \overset{f}{\longmapsto} e^{x+iy}$ $\qquad Re(f);\ Im(f) = ?$

$$e^{x+iy} = e^{x} \cdot e^{i\cdot y} = e^{x} \cdot (\cos y + i\sin y) =$$

$$\overset{\text{Eul. } \varphi = y}{=} \underbrace{e^{x}\cos y}_{u} + i \cdot \underbrace{e^{x}\sin y}_{v}$$





$e^{x}(\cos y + i\sin y)$

Elemi függvények

exp ✓

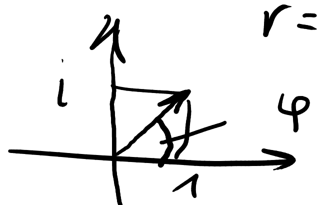
$$\sin z \overset{def}{=} \frac{e^{iz} - e^{-iz}}{2i} \ ;\ sh\, z = \frac{e^{z} - e^{-z}}{2}$$

$$\cos z \overset{def}{=} \frac{e^{iz} + e^{-iz}}{2} \ ;\ ch\, z = \frac{e^{z} + e^{-z}}{2}$$

$$\ln z \overset{def}{=} \underline{\overset{\in \mathbb{R}}{\ln(r)} + i\cdot\varphi + k\cdot 2\pi i}$$

$$z = r \cdot e^{i\varphi}$$

(3) $e^{z} = 1 + i \quad // \ln$



$r = \sqrt{2}$

$\varphi = \frac{\pi}{4}$

$$z = \ln(1+i) = \ln(\sqrt{2}) + i\frac{\pi}{4} + k 2\pi i$$

k egész