Gyakorlás:

$\ln z = \ln r + i\varphi + 2\pi i \cdot k$
$(z = r \cdot e^{i\varphi})$ kegyén

2 z 7
kéi zé hét

① $e^{2z} + ie^{z} + 2 = 0$

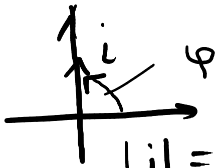
$w = e^{z}$

$w^2 + iw + 2 = 0;\quad w_{12} = \frac{-i \pm \sqrt{-1 - 4\cdot 1\cdot 2}}{2} =$

$= \frac{-i \pm \sqrt{-9}}{2} = \frac{-i \pm 3i}{2} \begin{cases} i \\ -2i \end{cases}$

$e^{z} = i$

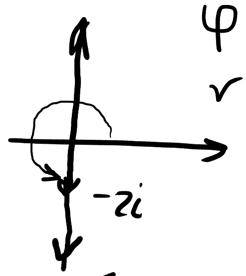
$z = \ln i$


$\varphi = \frac{\pi}{2}$
$|i| = 1$

$z = \underbrace{\ln 1}_{0} + i\frac{\pi}{2} + 2\pi i k = i\frac{\pi}{2} + 2\pi i k$ //

$e^{z} = -2i$

$z = \ln(-2i)$


$-2i$

$\varphi = \frac{3}{2}\pi$
$r = |-2i| = 2$

$z = \ln 2 + i\frac{3}{2}\pi + 2\pi i k$ //

② $\sin z = i$ $\quad ?\, z \in \mathbb{C}$ $\quad e^{-iz} = \frac{1}{e^{iz}}$

$\frac{e^{iz} - e^{-iz}}{2i} = i \quad / \cdot 2i$

$e^{iz} - e^{-iz} = -2$

$\sqrt{8} = \sqrt{4\cdot 2} = 2\sqrt{2}$

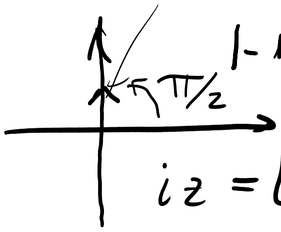
$w = e^{iz}$

$w - \frac{1}{w} = -2 \quad / \cdot w$

$w^2 - 1 + 2w = 0;\quad w^2 + 2w - 1 = 0$

$w_{1,2} = \frac{-2 \pm \sqrt{4 - 4\cdot 1\cdot(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

$e^{iz} = -1 + \sqrt{2} \quad / \ln$


$\pi/2$

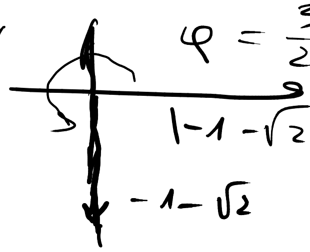
$|-1+\sqrt{2}| = -1 + \sqrt{2}$

$\boxed{\frac{1}{i} = \frac{1}{i}\cdot\frac{i}{i} = -i}$

$iz = \ln(-1+\sqrt{2}) + i\frac{\pi}{2} + 2\pi i k \quad / : i$

$z_1 = -i\ln(-1+\sqrt{2}) + \frac{\pi}{2} + 2\pi k$

$e^{iz} = -1 - \sqrt{2}$


$\varphi = \frac{3}{2}\pi$
$|-1-\sqrt{2}| = 1 + \sqrt{2}$
$-1-\sqrt{2}$

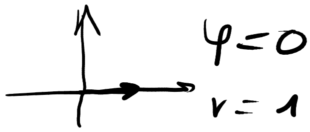
$i z_2 = \ln(1+\sqrt{2}) + i\frac{3}{2}\pi + 2\pi i k \quad / : i$

$z_2 = (-i)\cdot\ln(1+\sqrt{2}) + \frac{3}{2}\pi + 2\pi k$ //

③ $\sin z = 0$

$$\frac{e^{iz}-e^{-iz}}{2i}=0 \Rightarrow e^{iz}=e^{-iz} \quad /\cdot e^{iz}$$

$$\Rightarrow e^{2iz}=1 \qquad 2iz = \ln_{\mathbb{C}} 1 = \underbrace{\ln_{\mathbb{R}} 1}_{0} + i0 + 2\pi i k$$



$$\cancel{2}iz = \cancel{2}\pi i k$$

$$z = k\cdot\pi$$

Komplex differenciálhatóság

$[(z^2)' = 2z\ ;\ (\cos z)' = -\sin z, \ldots]$

1.) megj.: $f(x,y) = \begin{pmatrix} u(x,y) \\ v(x,y) \end{pmatrix}$

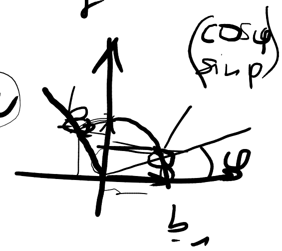
Nagy ~~Adilad~~

$\partial_x u = u'_x$

$$J^f(x,y) = \begin{bmatrix} \partial_x u & \partial_y u \\ \partial_x v & \partial_y v \end{bmatrix}$$

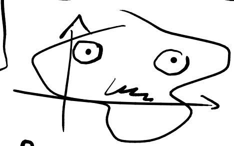
$\mathbb{R}^2$-beli diff. hatóság

$z \mapsto z\cdot w$



2.) $\mathbb{C} = \{ M \in \mathbb{R}^{2\times 2} \mid M \text{ forgatás nyújtás} \}$

$$\begin{bmatrix} L(\underline{b}_1) & L(\underline{b}_2) \end{bmatrix} = \begin{bmatrix} r\cos\varphi & -r\sin\varphi \\ r\sin\varphi & r\cos\varphi \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \cong a+ib$$



Tétel $f: \mathbb{C} \rightarrowtail \mathbb{C}$; $x_0+iy_0 \in \operatorname{int} \operatorname{Dom}(f)$; $f = u+iv$ ($u, v \in \mathbb{R}^2 \to \mathbb{R}$)

$$f \in \operatorname{Diff}_{\mathbb{C}}(x_0+iy_0) \iff \begin{cases} u, v \in \operatorname{Diff}_{\mathbb{R}^2}(x_0,y_0) \\ \text{és} \\ J^f(x_0,y_0) \in \mathbb{C} \end{cases}$$

Cauchy–Riemann-egyenletrendszer

$$\boxed{\begin{array}{l} \text{I.)}\ \partial_x u = \partial_y v \\ \text{II.)}\ \partial_y u = -\partial_x v \end{array}}$$

$a^2-b^2=(a+b)(a-b)$

④ Igazoljuk, hogy $z \xmapsto{f} z^2$ komplex differenciálható és $(z^2)' = 2z$.

$$z^2 = (x+iy)^2 = x^2 + 2xiy + \underbrace{(iy)^2}_{-y^2} = \underbrace{x^2-y^2}_{u} + i\underbrace{2xy}_{v}$$

$\checkmark$ u, v $\mathbb{R}^2$-ben valósan diff.

$$\begin{cases}\partial_x u = \partial_y v \\ \partial_y u = -\partial_x v\end{cases} \iff \begin{cases} 2x = 2x \\ -2y = -2y\end{cases} \checkmark$$

minden $x+iy$ komplex számra teljesül

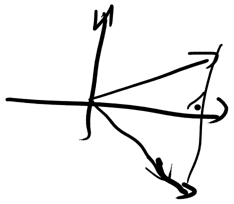
$$\boxed{f'(z) =} \begin{bmatrix}\partial_x u & \partial_y u\\ \partial_x v & \partial_y v\end{bmatrix} = \begin{bmatrix} a & -b \\ b & a\end{bmatrix} \cong a+ib = \boxed{\partial_x u + i\partial_x v}$$

$$f'(z) = 2x + i\cdot 2y = 2\cdot(x+iy) = 2z$$

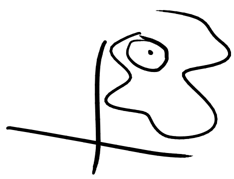
⑤ $z \xmapsto{f} \bar z$ diff.ható-e komplex értelemben?

$$\bar z = \overline{(x+iy)} = x - iy = \underbrace{x}_{u} + i\underbrace{(-y)}_{v}$$

$$\begin{cases} 1 = -1 \\ 0 = 0 \end{cases} \Rightarrow \text{ sehol sem differenciálható}$$



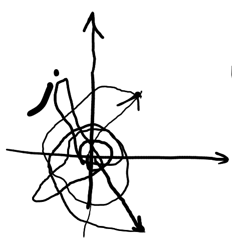
Borzasztó függvények



$\bar z$; $\operatorname{Re} z$; $\operatorname{Im} z$; $|z|$

Def: Legyen $f: \mathbb{C} \supset\!\!\to \mathbb{C}$; $z_0 \in \operatorname{int} \operatorname{Dom}(f)$; $w \in \mathbb{C}$.
Azt mondjuk, hogy f komplex deriválható a z_0 pontban, ha

$$\exists \lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0} = w$$



$(f'(z_0) = w)$

⑥ $f(z) = z\cdot\bar z$; $f'(0) = ?$

$z_0 = 0$

$$\lim_{z\to 0}\frac{z\cdot\bar z - 0\cdot\bar 0}{z-0} = \lim_{z\to 0}\frac{\cancel{z}\cdot\bar z}{\cancel{z}} = \lim_{z\to 0}\bar z = 0$$

$$f(z)=z\cdot\overline{z} = (x+iy)\cdot(x-iy) = x^2-\underbrace{(iy)^2}_{-(y^2)} = \underbrace{x^2+y^2}_{u} + \underbrace{0\cdot i}_{v}$$

$$\begin{cases} \partial_x u = \partial_y v \\ \partial_y u = -\partial_x v \end{cases} \iff \begin{cases} 2x = 0 \\ 2y = -0 \end{cases} \iff x=0 \text{ és } y=0$$

→ u, v valósan diff.

→ C-R-e.r. csak a 0-ban teljesül