Komplex függvénytan

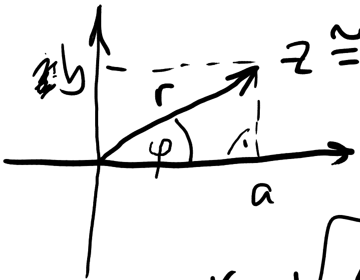
Komplex számok

$$a+bx \;\Big|\; x^2 = 1\cdot(x^2+1)\underbrace{-1}, \quad x^2 \cong -1$$

$$\mathbb{C} = \{a+ib \mid a,b \in \mathbb{R}\} \cong \mathbb{R}[x]/(x^2+1) \cong$$

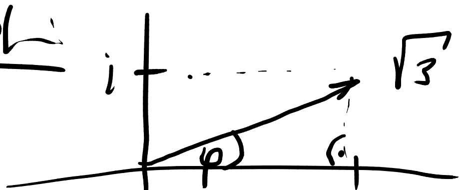
$$i^2 = -1 \qquad \cong \{M \in \mathbb{R}^{2\times 2} \mid M \text{ forgatva nyújtás}\}$$

$$(a,b) \cong a+ib$$



$z \cong a+ib$

Pl:



$\sqrt{3}+i \qquad r=2 \qquad \varphi = \frac{\pi}{6}$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$Re(a+ib) = a \in \mathbb{R}$

$Im(a+ib) = b \in \mathbb{R}$

$$tg\,\varphi = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \to \varphi = \frac{\pi}{6}$$

- algebrai alak: $z = a+bi = r\cdot\cos\varphi + ir\sin\varphi$
- trigonometrikus alak: $z = r\cdot(\cos\varphi + i\sin\varphi)$
- exponenciális alak $z = re^{i\varphi}$ $\qquad \varphi \in \mathbb{R}$

Euler-formula:

$$\boxed{e^{i\varphi} = \cos\varphi + i\sin\varphi}$$

$$z_1\cdot z_2 = r_1 r_2 (\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2))$$
$$= r_1 e^{i\varphi_1}\cdot r_2 e^{i\varphi_2} = r_1 r_2 e^{i\varphi_1 + i\varphi_2} = r_1 r_2 e^{i(\varphi_1+\varphi_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\cdot e^{i(\varphi_1-\varphi_2)}$$

$$z^n = (r\cdot e^{i\varphi})^n = r^n\cdot e^{in\varphi}$$

Komplex függvények

$f: \mathbb{C} \to \mathbb{C}$; $x+iy \mapsto f(x+iy)$

$\mathrm{Dom}(f) \subseteq \mathbb{C}$

$f(x+iy) = \underbrace{u(x,y)} + i\cdot \underbrace{v(x,y)}$

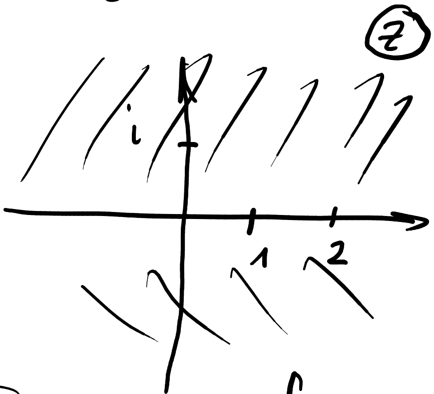
$\mathrm{Re}(f): \mathbb{R}^2 \to \mathbb{R}$ $\qquad$ $\mathrm{Im}(f): \mathbb{R}^2 \to \mathbb{R}$

(1) $f(z) = z^2$

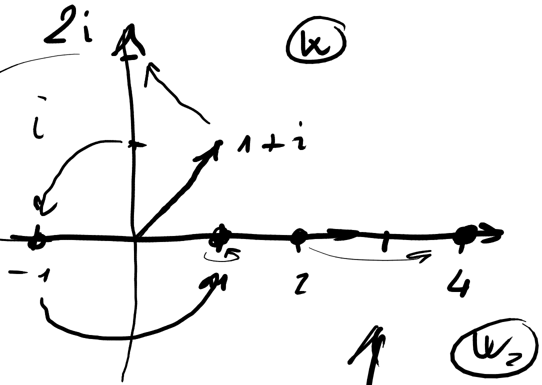
Trükk: $z = x+iy$

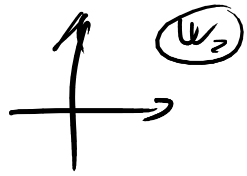
$(x+iy)^2 = x^2 + 2xiy + \underbrace{(iy)^2}_{-1\cdot y^2} = x^2 + 2xiy - y^2 =$

$= \underbrace{x^2 - y^2}_{u} + i\cdot\underbrace{(2xy)}_{v}$



$(1+i)^2 =$
$= 1^2 + 2i + i^2 =$
$= 1 + 2i - 1$
$= 2i$





(3) $x+iy \xmapsto{f} e^{x+iy}$

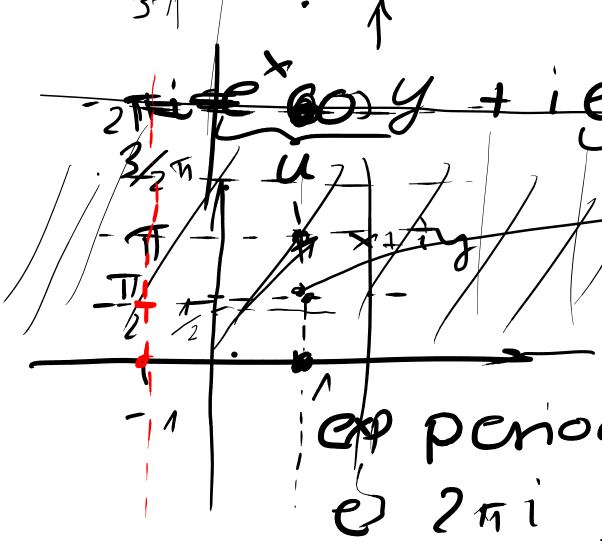
u, v = ?

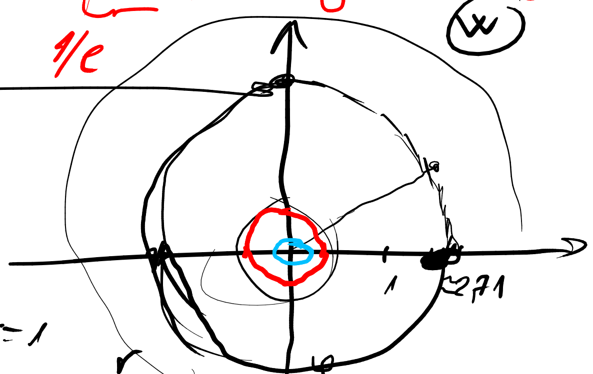
$\varphi = y$

$e^{x+iy} = e^x \cdot e^{iy} = e^x \cdot (\cos y + i \sin y) =$

$\underbrace{e^x \cos y}_{u} + i\,\underbrace{e^x \sin y}_{v}$

$e^{-1}\cdot(\cos y + i \sin y)$





exp periodikus

$e^{2\pi i}$ [illegible] egy periódus

$x = 1$

$e^x \cdot (\cos y + i \sin y)$

Elemi függvények

(polinom, trig, hip, exp, log)

$$\sin z \stackrel{def}{=} \frac{e^{iz}-e^{-iz}}{2i} \quad ; \quad \operatorname{sh} z = \frac{e^{z}-e^{-z}}{2}$$

$$\cos z \stackrel{def}{=} \frac{e^{iz}+e^{-iz}}{2} \quad ; \quad \operatorname{ch} z = \frac{e^{z}+e^{-z}}{2}$$

[:) HF $e^{iz} = \cos z + i \sin z$]

$$\rightarrow \operatorname{Ln} z \stackrel{def.}{=} \ln(\underset{}{\overset{\in\mathbb{R}}{r}}) + i\cdot\varphi + k\cdot 2\pi i$$

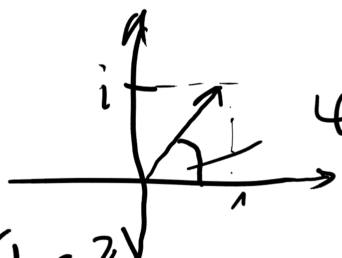
$z = re^{\varphi}$ k egész

4)

$$e^{z} = 1+i \quad / \ln$$

[:) HF $e^{z} = \frac{1}{2}i + \frac{\sqrt{3}}{2}$]

$$z = \ln(1+i)$$



$\varphi = \frac{\pi}{4}$

$$z = \ln\sqrt{2} + i\frac{\pi}{4} + k\cdot 2\pi i$$

k egész ($k \in \mathbb{Z}$)

$$r = |1+i| = \sqrt{1^2+1^2} = \sqrt{2}$$